Light Source II

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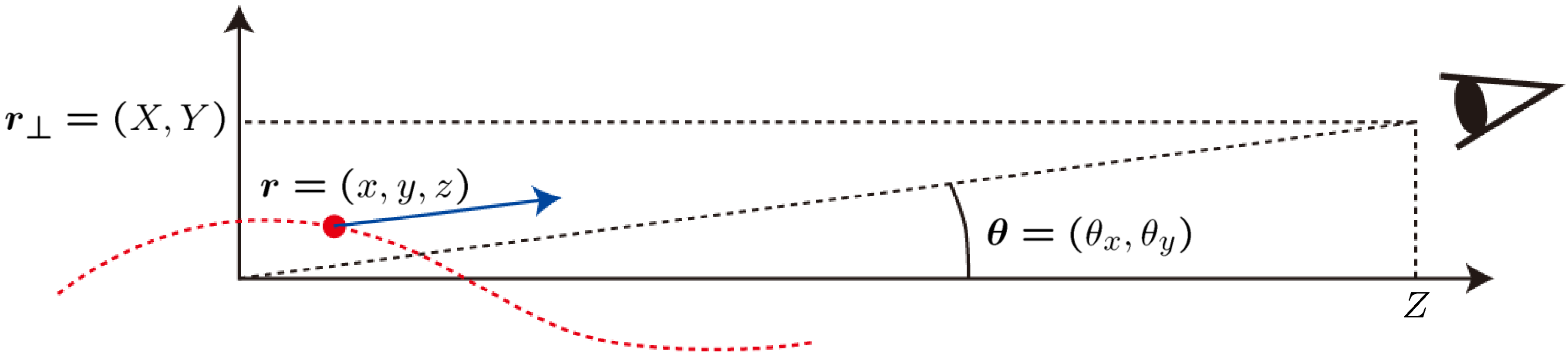
Characteristics of SR (2)

- Electron Trajectory in the ID
- Qualitative Description of Wiggler Radiation
- Qualitative Description of Undulator Radiation

Coordinate Systems



SR emitted by an electron moving at ***r*** = *(x,y,z)*
Observation of SR at ***R*** = *(X,Y,Z)*

If the far-field approximation (|***r***|<<*Z*) is applicable, the radiation pattern depends only on the observation angle $\boldsymbol{\theta} = (\theta_x, \theta_y)$.

Field Integrals

$$\frac{d\boldsymbol{P}}{dt} = m\gamma\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{v}\times\boldsymbol{B} \Rightarrow \begin{cases} m\gamma\dot{v_x} = -e(v_yB_z - v_zB_y) \\ m\gamma\dot{v_y} = -e(v_zB_x + v_xB_z) \end{cases}$$

Equation of motion of an electron moving in a magnetic field ***B***

$$\Downarrow B_z \equiv 0$$

$$m\gamma\frac{dv_{x,y}}{v_z dt} = m\gamma\frac{dv_{x,y}}{dz} = \pm eB_{y,x}$$

$$\boxed{\begin{aligned} \beta_{x,y} &= \pm\frac{e}{\gamma mc}\int^z B_{y,x}(z')dz' \equiv \pm\frac{e}{\gamma mc}I_{1y,1x}(z) \\ x, y &= \pm\frac{e}{\gamma mc}\int^z\int^{z'} B_{y,x}(z'')dz'' \equiv \pm\frac{e}{\gamma mc}I_{2y,2x}(z) \end{aligned}}$$

I_1, I_2 : 1st and 2nd field integrals of the ID

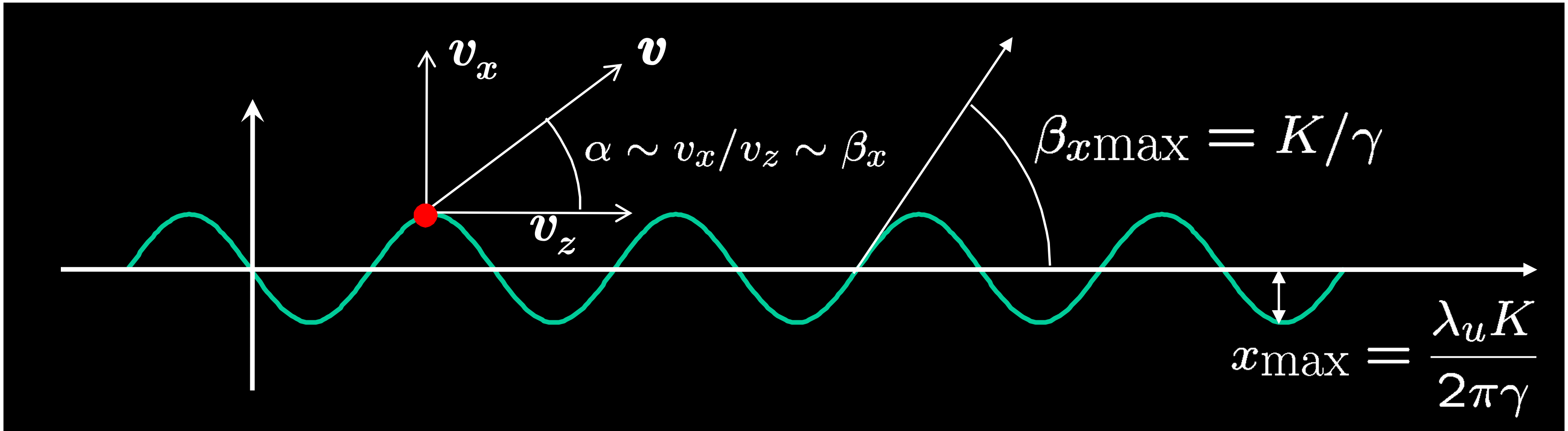
Trajectory in an Ideal ID

$$\begin{cases} B_x(z) = 0 \\ B_y(z) \sim B_0 \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \qquad \begin{cases} \beta_y = 0 \\ \beta_x = \frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \qquad \begin{cases} y = 0 \\ x = \frac{\lambda_u K}{2\pi\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases}$$

magnetic field ➡ velocity ➡ position

$$K = \frac{eB_0\lambda_u}{2\pi mc} = 93.37 B_0(\mathrm{T})\lambda_u(\mathrm{cm})$$

K value, Deflection parameter



E_e=8GeV,K=1,λ_u=5cm : β_{xmax}=64μrad,x_{max}=0.5μm

Effects due to the ID Magnetic Field

transverse velocity

$$\beta_x = \frac{K}{\gamma}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

longitudinal velocity

$$\beta_z = \sqrt{\beta^2 - \beta_x^2}$$

$$= \underbrace{1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}}_{\bar{\beta}_z\text{ :average velocity}} - \underbrace{\frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)}_{\text{oscillating term}}$$

ID field induces:

- transverse(x) oscillation
- longitudinal (z) oscillation
- effective deceleration($\Delta\beta_z = K^2/4\gamma^2$)

General Form of Time Squeezing

$$\frac{d\tau}{dt} = 1 - \boldsymbol{\beta} \cdot \boldsymbol{n}$$

$$\boxed{\begin{aligned} \beta_z &= \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \\ &\sim 1 - (\gamma^{-2} + \beta_x^2 + \beta_y^2)/2 \\ n_z &\sim 1 - (\theta_x^2 + \theta_y^2)/2 \end{aligned}}$$

$$= \frac{1}{2\gamma^2} + (\theta_x - \beta_x)^2 + (\theta_y - \beta_y)^2$$

Time squeezing takes place most significantly when the direction of the electron motion coincides with that of observation ($\boldsymbol{\beta} = \boldsymbol{\theta}$).

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- Electron Trajectory in the ID
- Qualitative Description of Wiggler Radiation
- Qualitative Description of Undulator Radiation

Wiggler Radiation

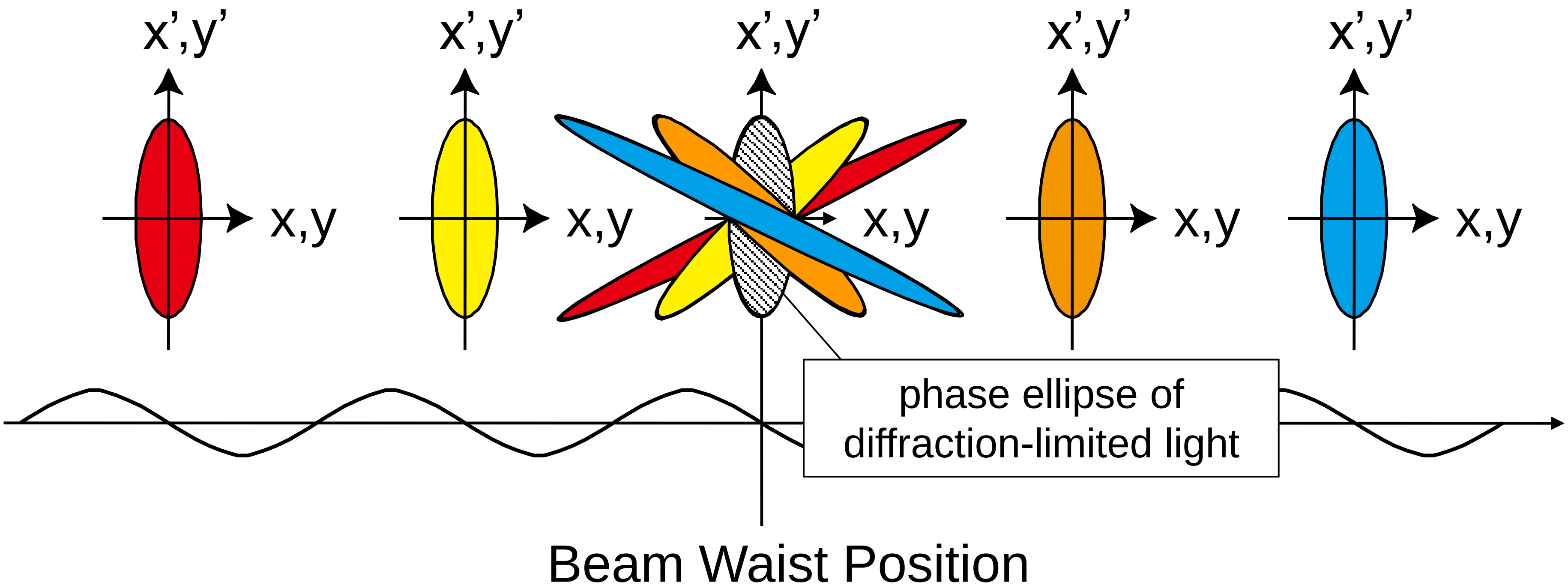
- Wiggler radiation (WR) is regarded as incoherent sum of SR at each position.
  - Summation as photons in the framework of geometrical optics.

Flux: $F_W \sim 2NF_{BM}$

Emittance: $\sigma_{x',y'} \times \sigma_{x,y} \gg \lambda/4\pi$

Brilliance: $B_W \ll 2NB_{BM}$

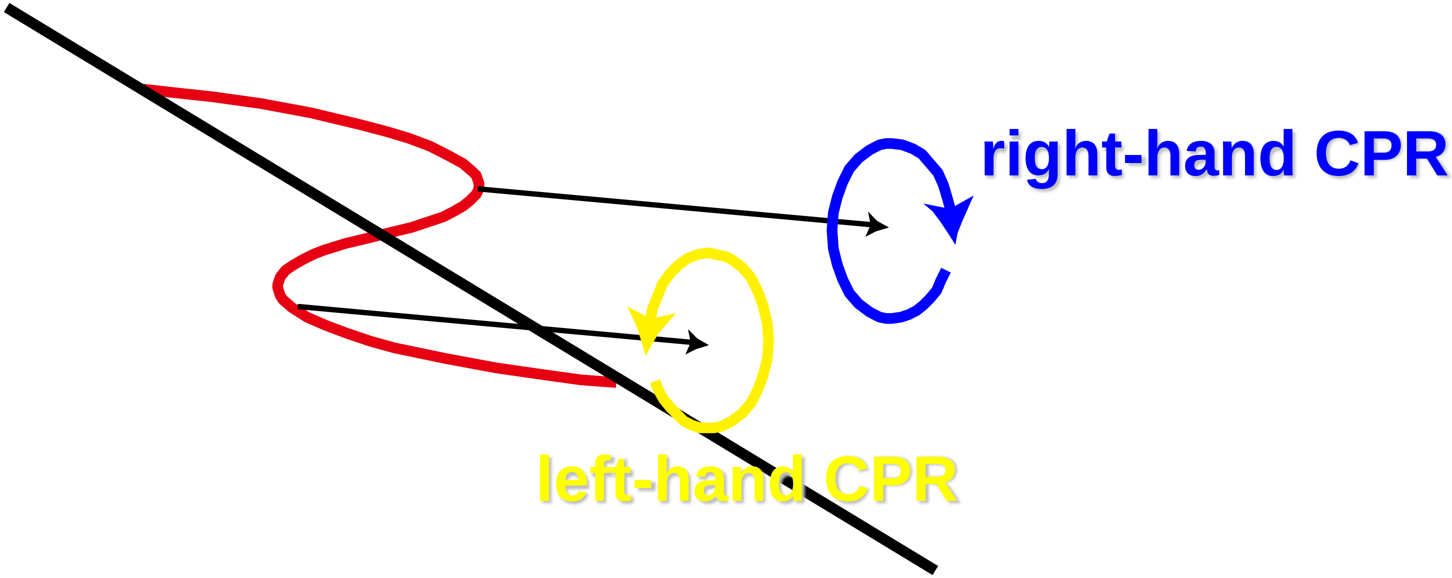
Photon Distribution in Phase Space



- Larger *N* results in larger area of photon distribution in the phase space, i.e., larger emittance.
- *B* does not linearly depend on *N*

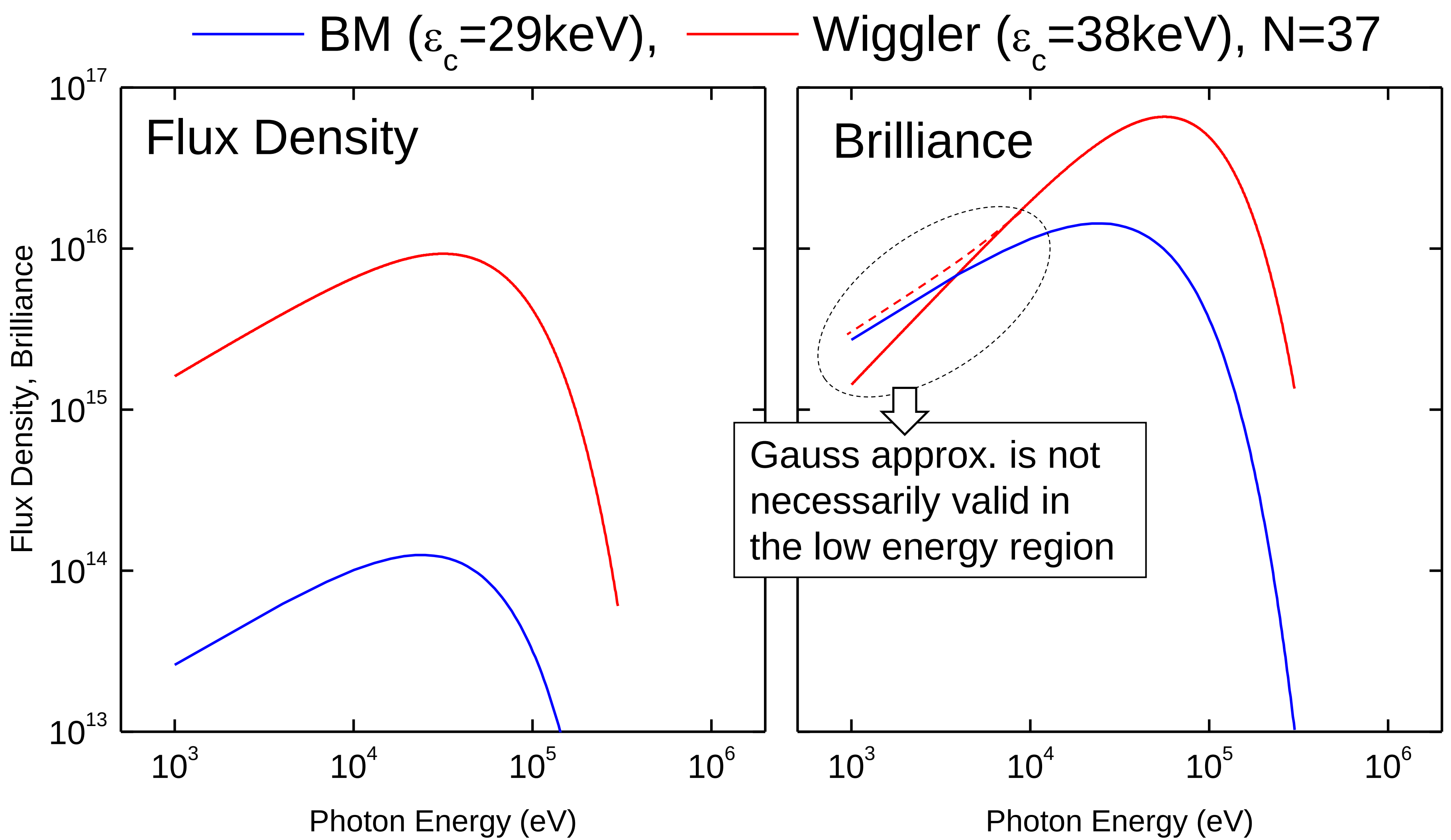
Polarization

- No circular polarized radiation (CPR) is observed unlike the BM radiation even off axis, due to cancellation of CPR components.



- EMPW is a special wiggler to utilize CPR by introducing a vertical motion.

Comparison with BM Radiation



Characteristics of SR (2)

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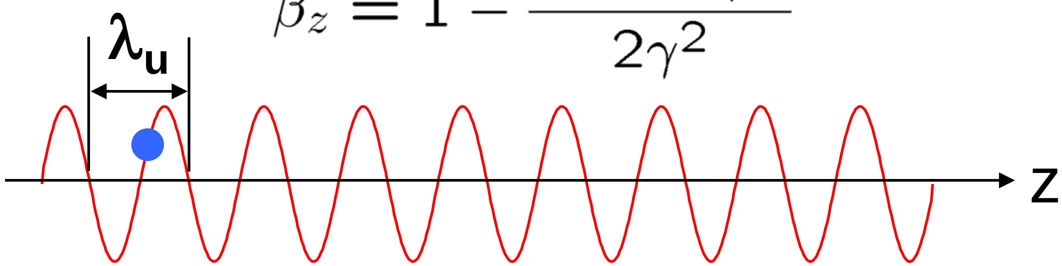
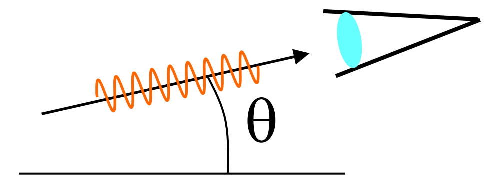
Fundamental Wavelength

$$\bar{\beta}_z = 1 - \frac{1 + K^2/2}{2\gamma^2}$$





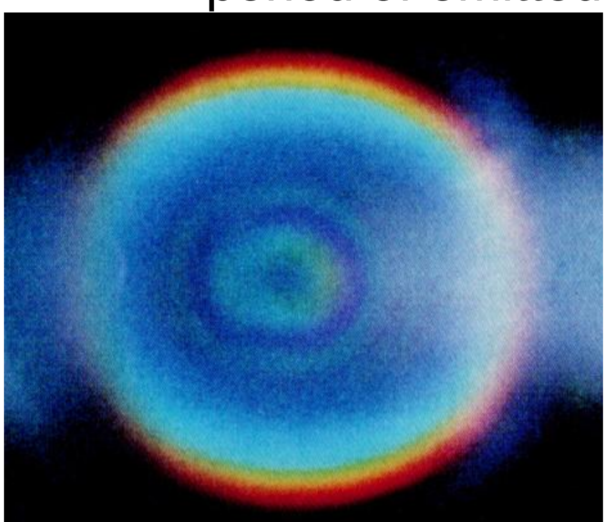
$$T = \lambda_u / v_z = \lambda_u / c$$

period of electron motion
= period of emitted light

time squeezing

$$T' = T(1 - \bar{\beta}_z \cos\theta)$$

period of observed light



H. Kitamura et al.,
J. Appl. Phys. 21 (1982) 1728

Fundamental Wavelength λ_1

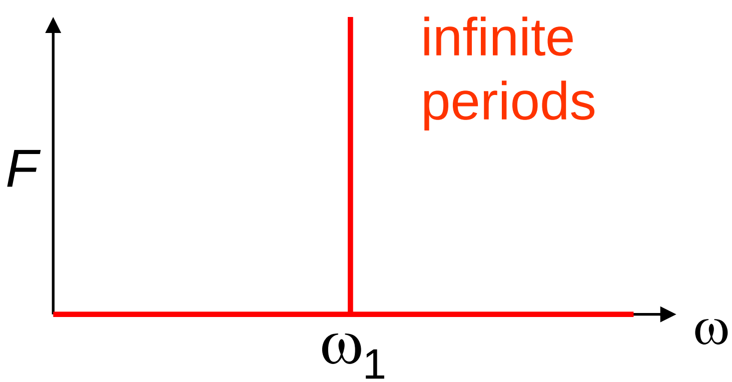
$$\begin{aligned} \lambda_1 &= \lambda_u (1 - \bar{\beta}_z \cos\theta) \\ &= \frac{\lambda_u}{2\gamma^2} (1 + \gamma^2\theta^2 + K^2/2) \\ \omega_1 &= 2\pi c / \lambda_1 \end{aligned}$$

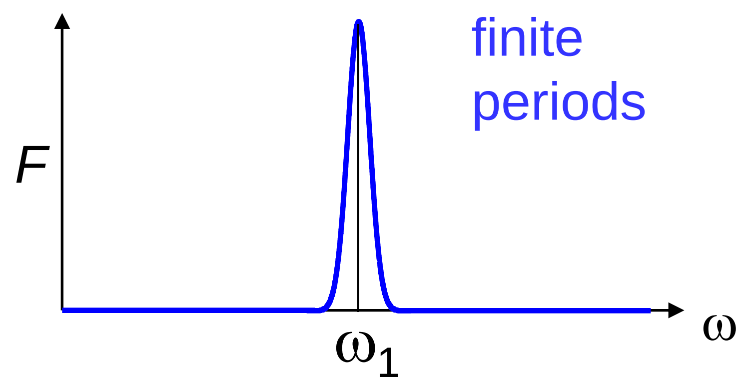
UR with Infinite Periods

- If the undulator length is infinite, the pulse duration is infinitely long, and thus the radiation is completely monochromatic with line spectrum.

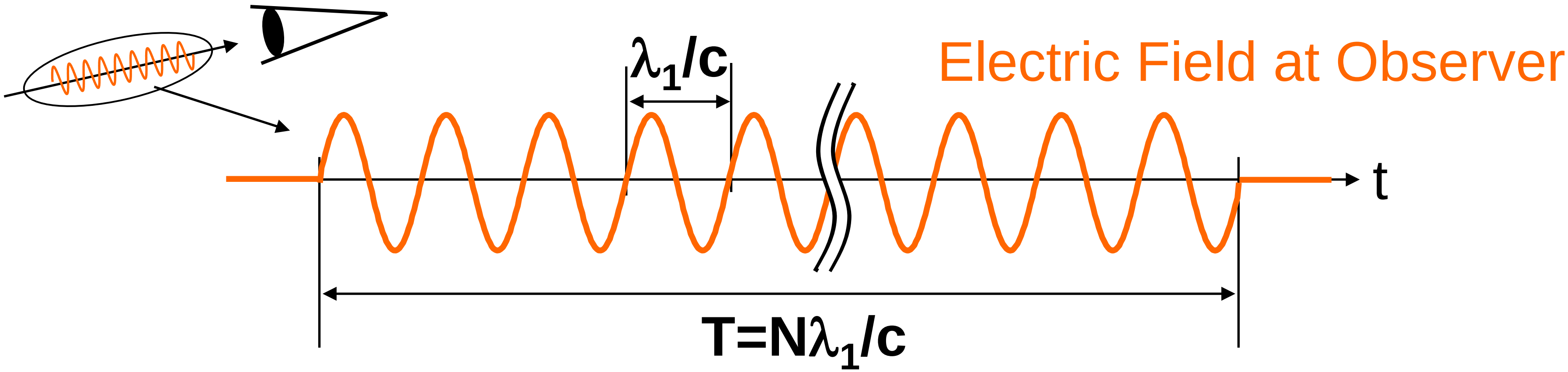
$$\frac{d^2F}{dx'dy'} \propto \delta(\omega - \omega_1) = \delta\left(\omega - \frac{4\pi c\gamma^2/\lambda_u}{1 + K^2/2 + \gamma^2\theta^2}\right)$$

- In practice, the undulator length is finite, so the line spectrum is broadened.





Effects due to Finite Periods



$$E(t) = \begin{cases} E_0 \sin \omega_1 t & ; -T/2 \leq t \leq T/2 \\ 0 & ; t < -T/2, T/2 < t \end{cases}, \quad \omega_1 = 2\pi c/\lambda_1$$

Fourier Transform

$$\frac{d^2F}{dx'dy'} \propto |\tilde{E}(\omega)|^2 \propto \mathrm{sinc}^2\left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)}\right]$$

Square of “sinc” function dominates the UR

Brief Note on UR Formulae

- In the previous derivations of UR spectral function, no knowledge on electro-dynamics is required.
- In practice, E_0 is a complicated function of θ and K, and needs to be calculated by Fourier transforming the electric field derived from the Lienard-Wiecherd potential.
- However, the simple derivation gives us a clear understanding on UR properties.

Energy and Angular Profile of UR

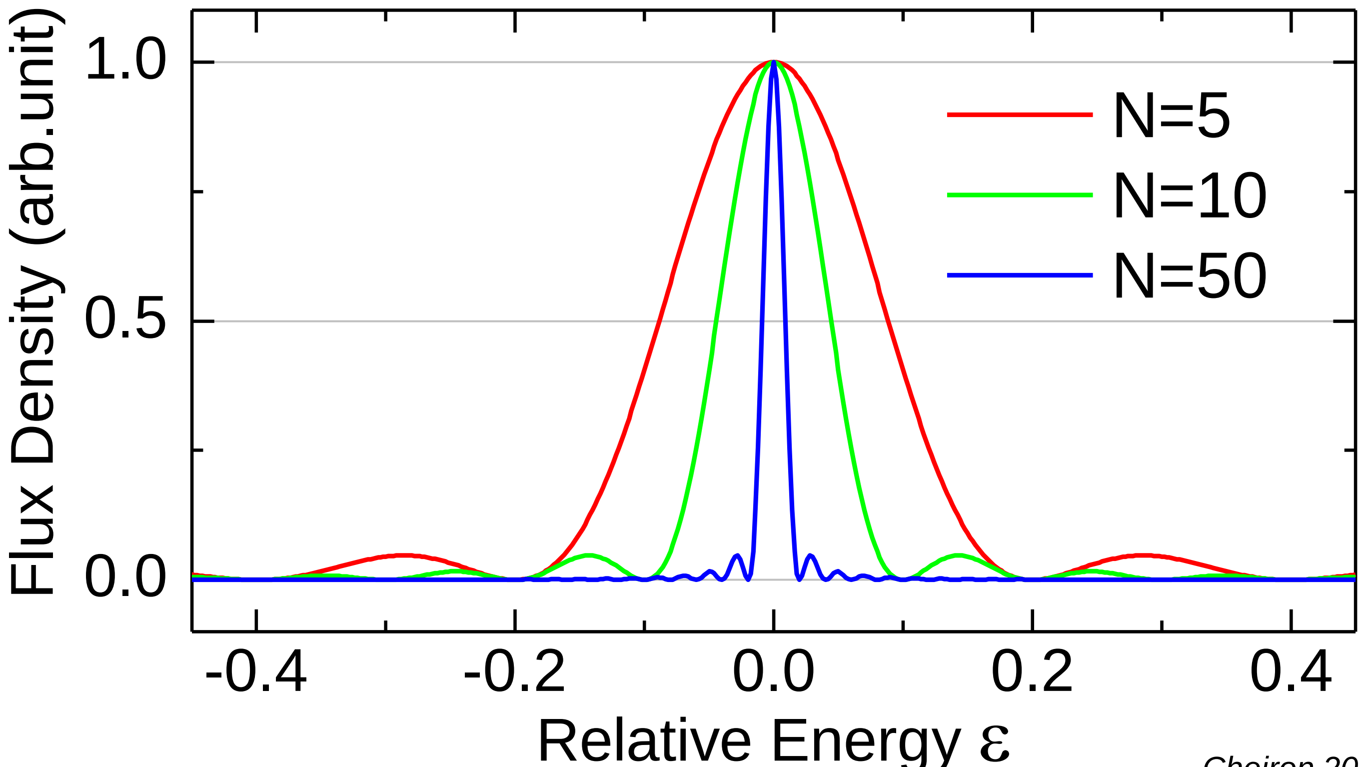
$$\frac{d^2F(\omega,\theta)}{d\Omega d\omega/\omega} = F_0 \mathrm{sinc}^2\left[\pi N \frac{\omega-\omega_1(\theta)}{\omega_1(\theta)}\right]$$

$$= \begin{cases} \text{Energy Profile at } \theta = 0 \\ F_0 \mathrm{sinc}^2(N\pi\varepsilon) \\ \quad ;\varepsilon = [\omega-\omega_1(0)]/\omega_1(0) \\ \text{Angular Profile at } \omega=\alpha\omega_1(0) \\ F_0 \mathrm{sinc}^2[N\pi(\alpha\Theta^2+\alpha-1)] \\ \quad ;\Theta = \gamma\theta/\sqrt{1+K^2/2} \end{cases}$$

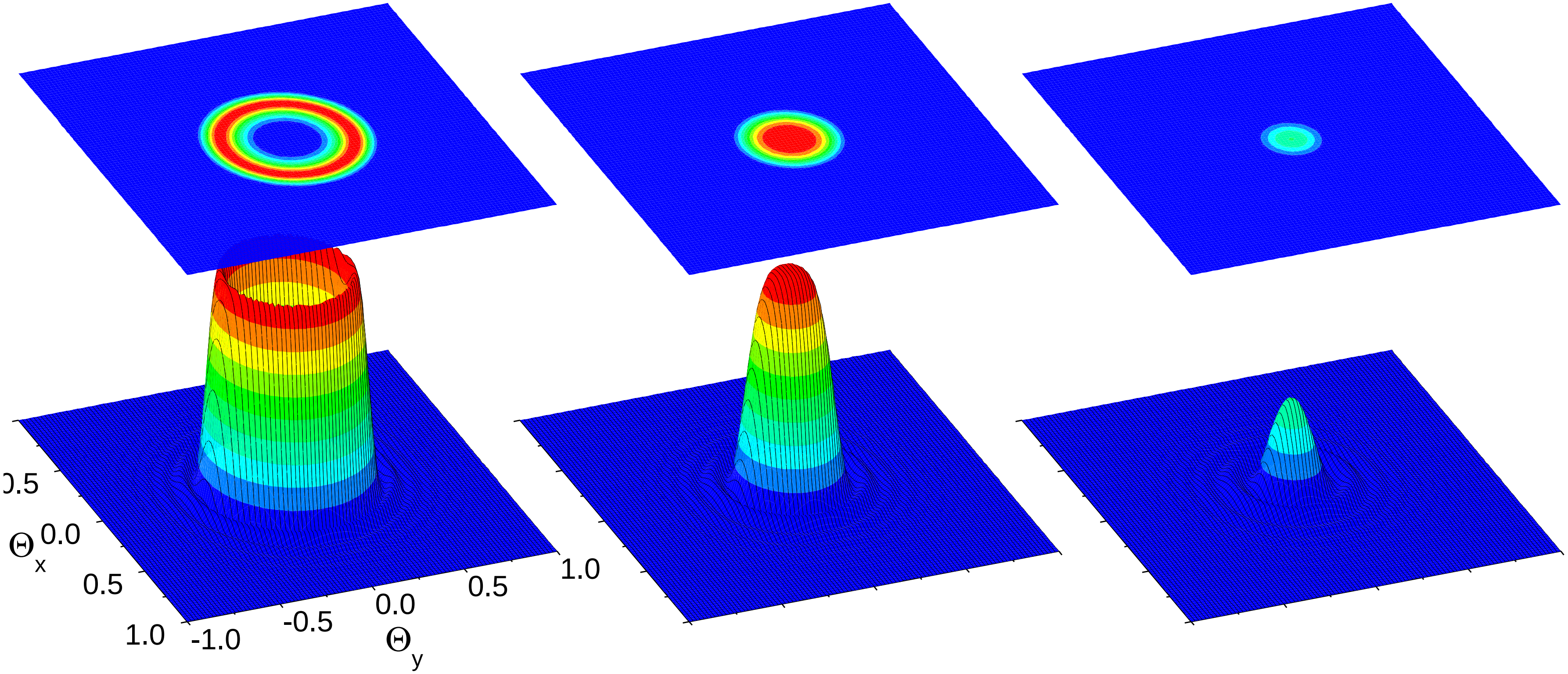
Energy Profile: Example

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2(N\pi\varepsilon); \quad \mathrm{sinc}^2(2.783) \sim 1/2$$

$$\Rightarrow \quad \left.\frac{\Delta\omega}{\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{N}$$



Angular Profile: Example



$\omega=0.9\omega_1(0)$

lower energy

$\omega=\omega_1(0)$

fundamental energy

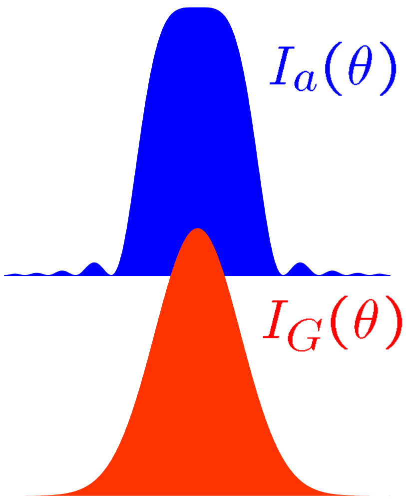
$\omega=1.05\omega_1(0)$

higher energy

Angular Divergence and Beam Size

Angular Profile at $\omega=\omega_1(0)$

$$I_a(\theta) = F_0 \mathrm{sinc}^2\left[\frac{\pi N(\gamma\theta)^2}{1+K^2/2}\right]$$



approximation

Gaussian Profile with $\sigma_{r'}$

$$I_G(\theta) = F_0 \exp(-\theta^2/2\sigma_{r'}^2)$$

$$\sigma_{r'} = \sqrt{\frac{1+K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

Angular Divergence of UR ($L=N\lambda_u$)

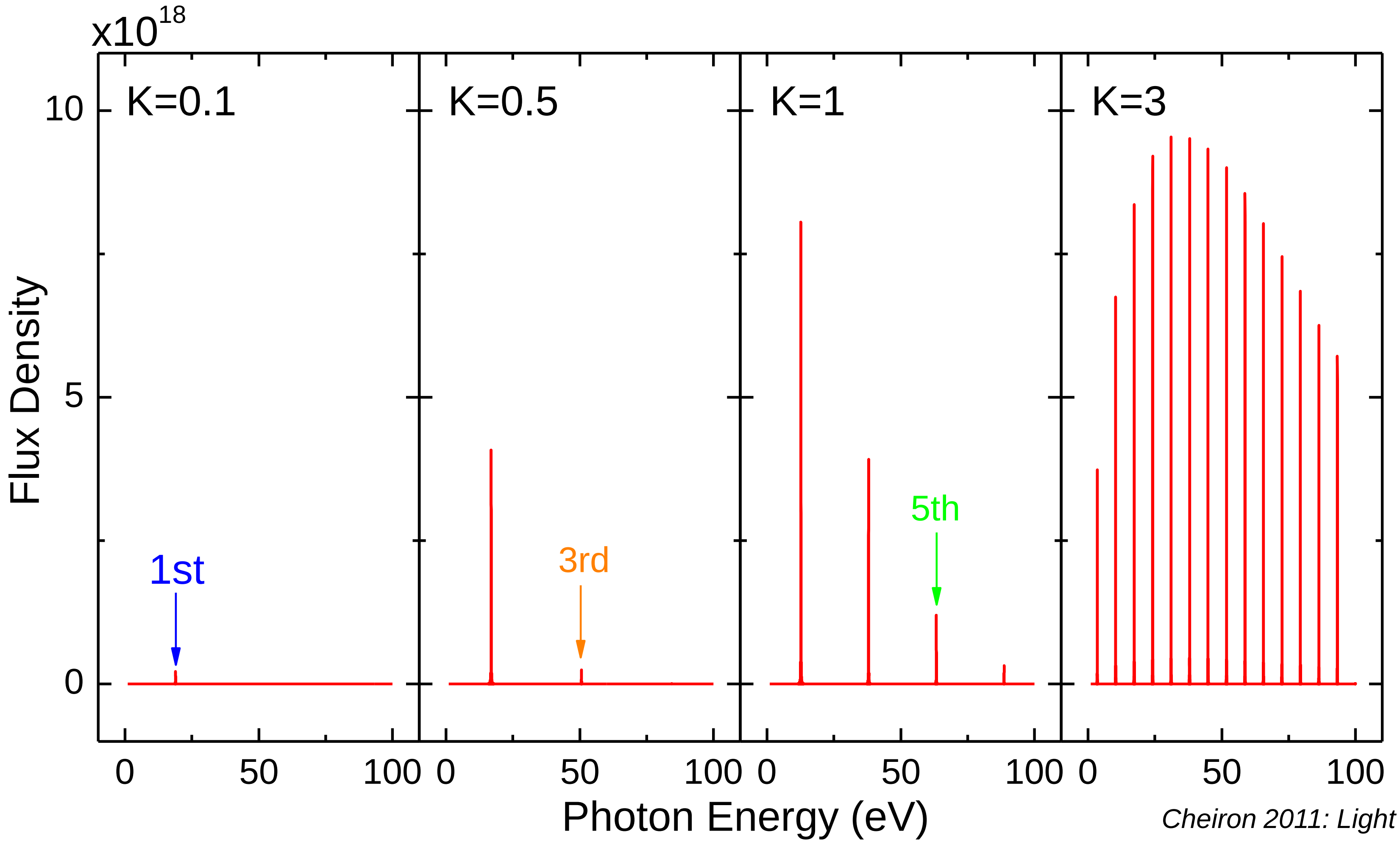
Diffraction Limit (UR is Spatially Coherent)

$$\sigma_r = \frac{\lambda_1}{4\pi\sigma_{r'}} = \frac{\sqrt{\lambda_1 L}}{4\pi}$$

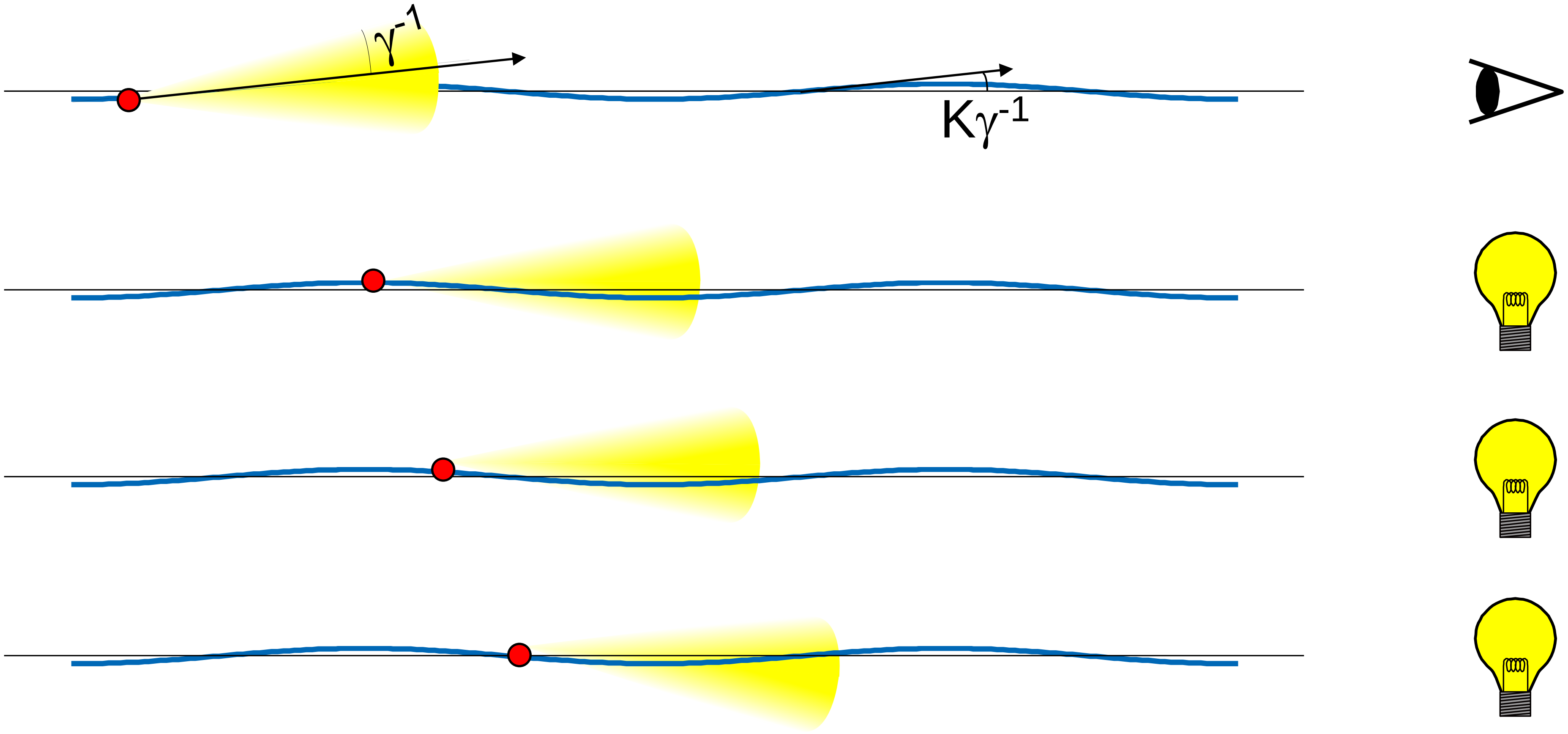
Source Size of UR

Higher Harmonics

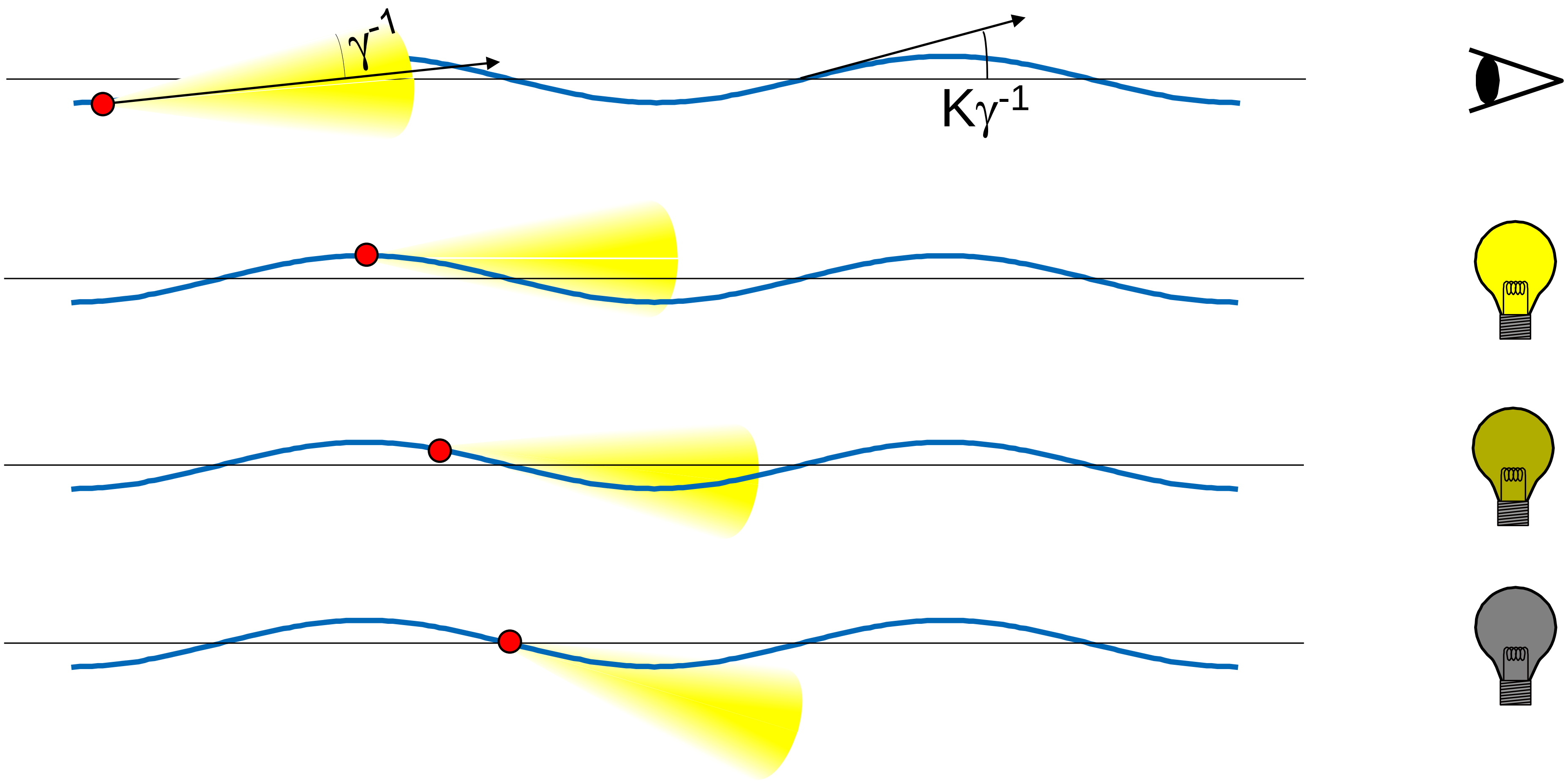
- In addition to ω_1, photons with the energy at $n\omega_1$ is also observed, where n is an integer.



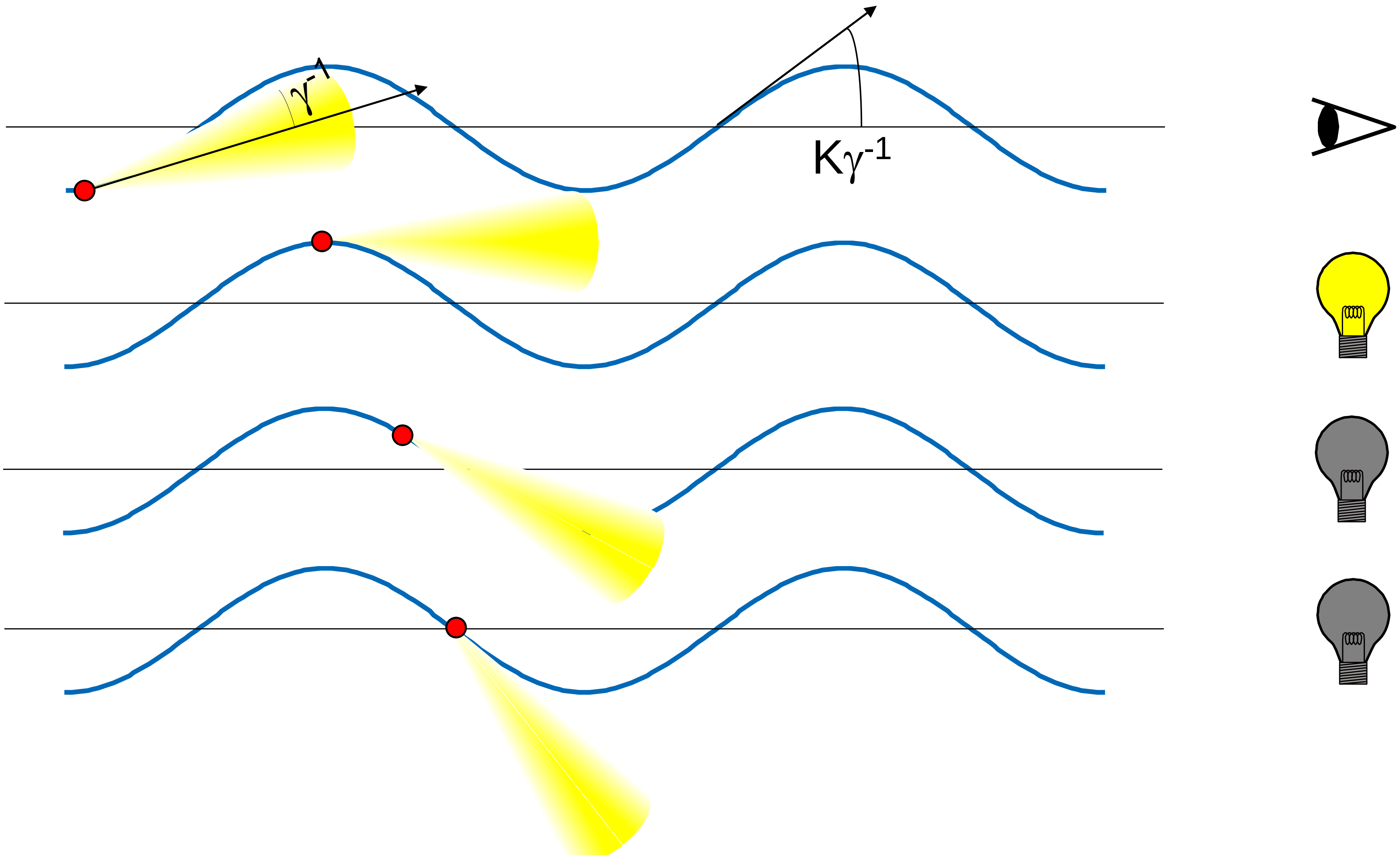
Observation of UR: K<<1 Case



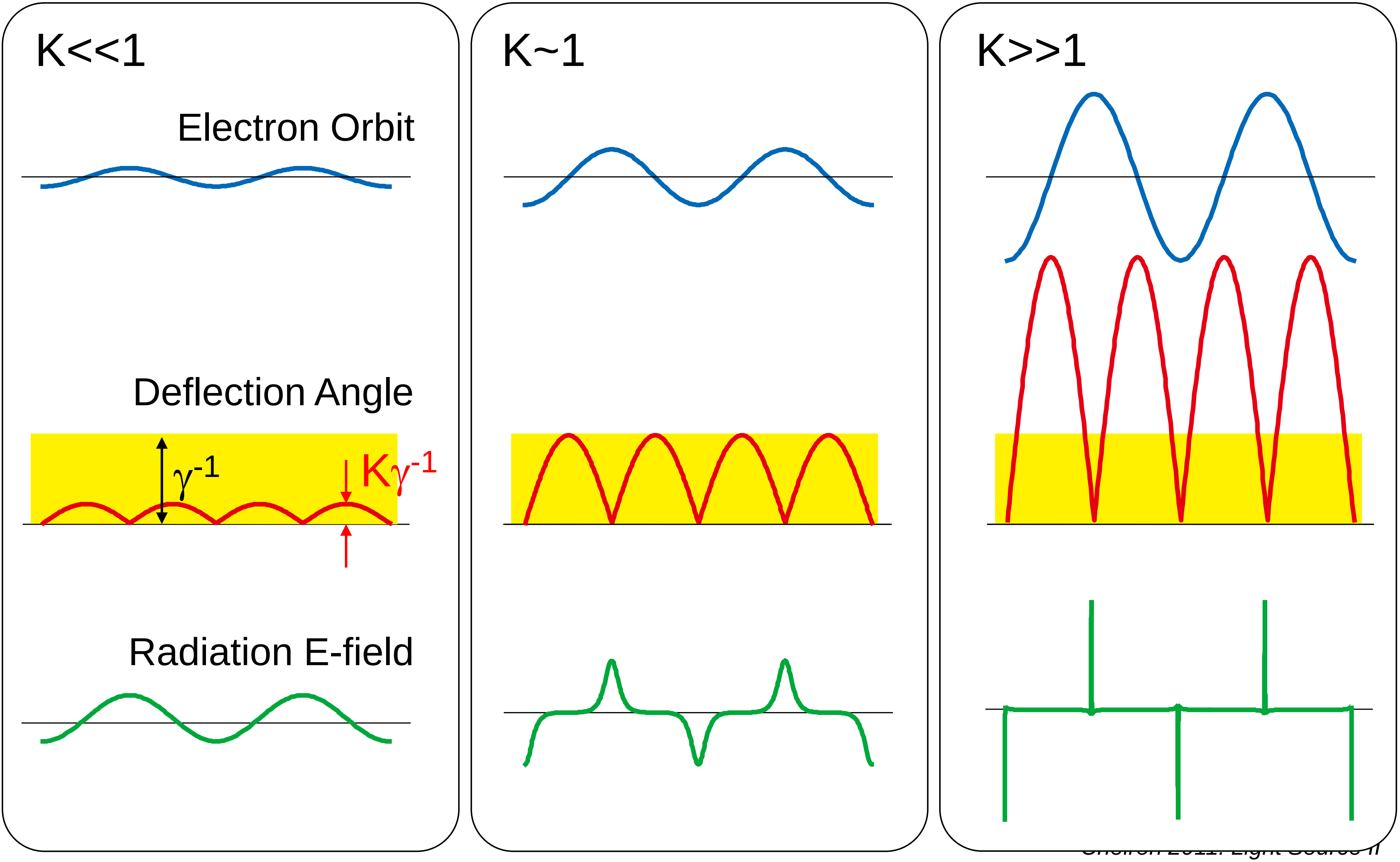
Observation of UR: K~1 Case



Observation of UR: K>>1 Case



Mechanisms of Higher Harmonics



Optical Properties of Higher Harmonics

For the n-th harmonic radiation,

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2 \left[\pi n N \frac{\omega - n\omega_1(\theta)}{n\omega_1(\theta)} \right]$$

$$\left.\frac{\Delta\omega}{n\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{nN}$$ band width

$$\sigma_{r'n} = \sqrt{\frac{1+K^2/2}{4nN\gamma^2}} = \sqrt{\frac{\lambda_1/n}{2L}}$$ angular divergence

$$\sigma_{rn} = \frac{\lambda_1/n}{4\pi\sigma_{r'n}} = \frac{\sqrt{L\lambda_1/n}}{4\pi}$$ Source size

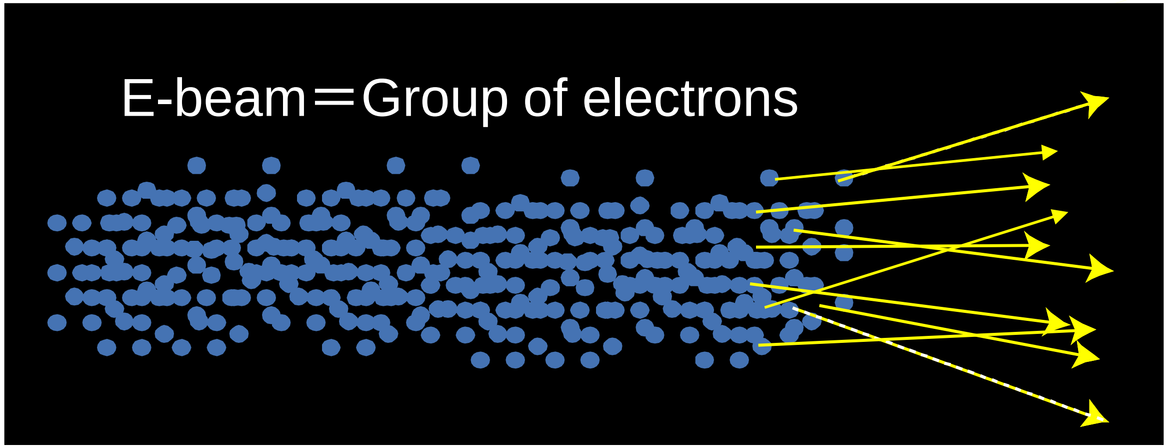
Practical Knowledge on Undulator Radiation

"Practical Knowledge" on UR

- The undulator is a "double-edged sword"!
  - Much higher brilliance is available than the BM and wiggler, *if appropriately used*.
  - High heat load on optical elements
  - Quasi-monochromatic and small angular spread imposes an accurate adjustment of BL components.
- Practical knowledge for the utilization of UR
  - Effects due to the electron beam quality
  - Simple evaluation of optical properties
  - Heat load reduction
  - …….

Electron Beam Quality (1)

- The property of UR from a single electron is similar to a laser
  - Small Size & Angular Divergence, Narrow Bandwidth
- In practice, UR in the beamline is emitted by the beam comprising a huge number of electrons.
- These electrons have different positions, angle, and energies.



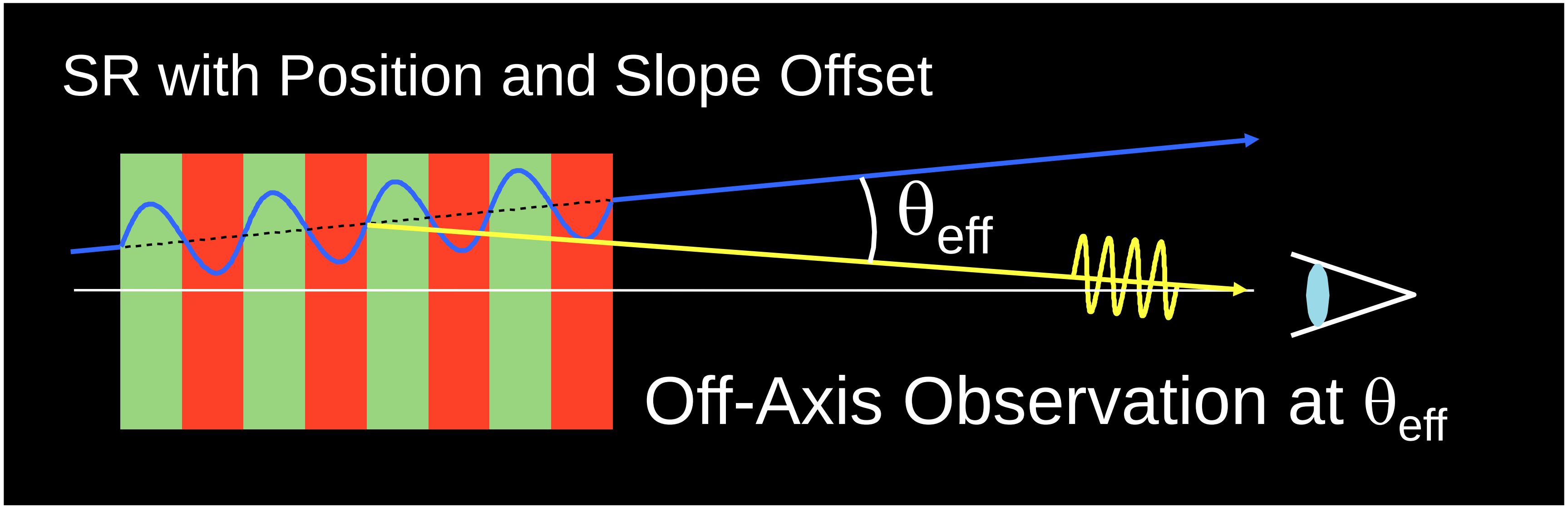
The performance of UR depends largely on the e-beam quality

Electron Beam Quality (2)

- Electron-beam parameters related to the performance of UR
  - –Beam Size $\sigma_x,\sigma_y \rightarrow$ Source Size
  - –Angular Divergence $\sigma_{x'},\sigma_{y'} \rightarrow$ Directivity
    - ➢ The minimum value of the product $\sigma_x\sigma_x$ and $\sigma_y\sigma_y$ are called the electron beam emittances in the x and y directions.
  - –Energy Spread $\sigma_E/E \rightarrow$ Monochromaticity

Effects due to Finite Emittance (1)

- Effects due to Finite Emittance of the Electron Beam
  - – Injection to the undulator with angular and positional offset

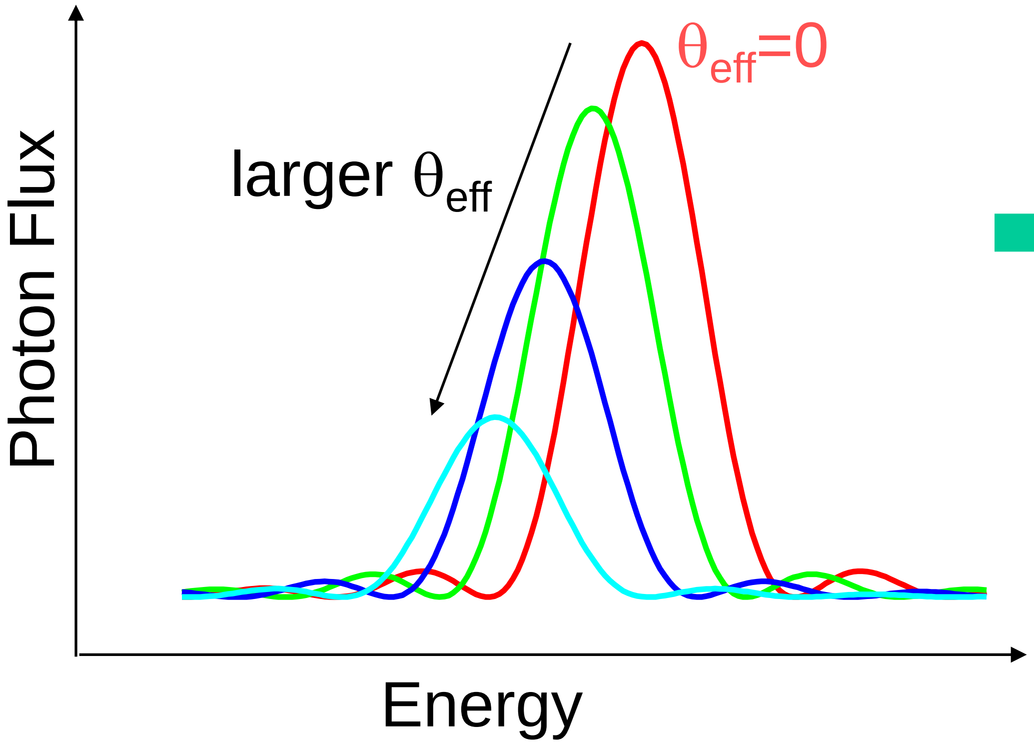


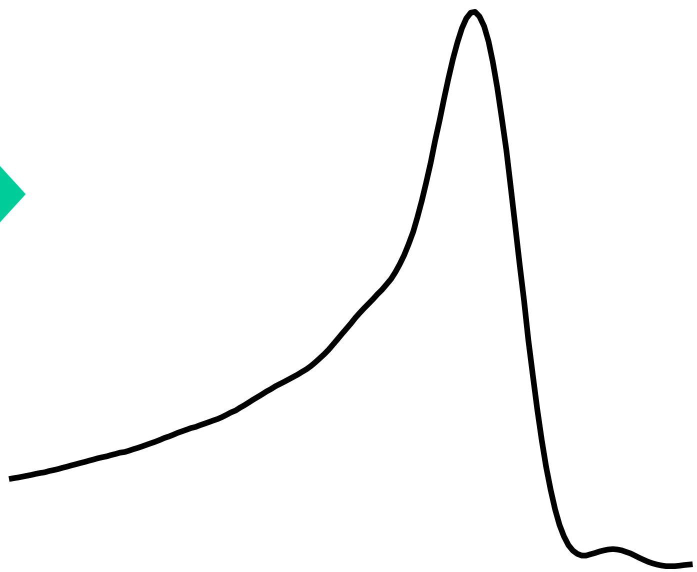
Effects due to Finite Emittance (2)

Off-axis observation at θ_{eff}

Peak shift to lower energy

$$\omega_1(\theta) = \frac{4\pi c\gamma^2/\lambda_u}{1 + \boxed{\gamma^2\theta^2} + K^2/2}$$



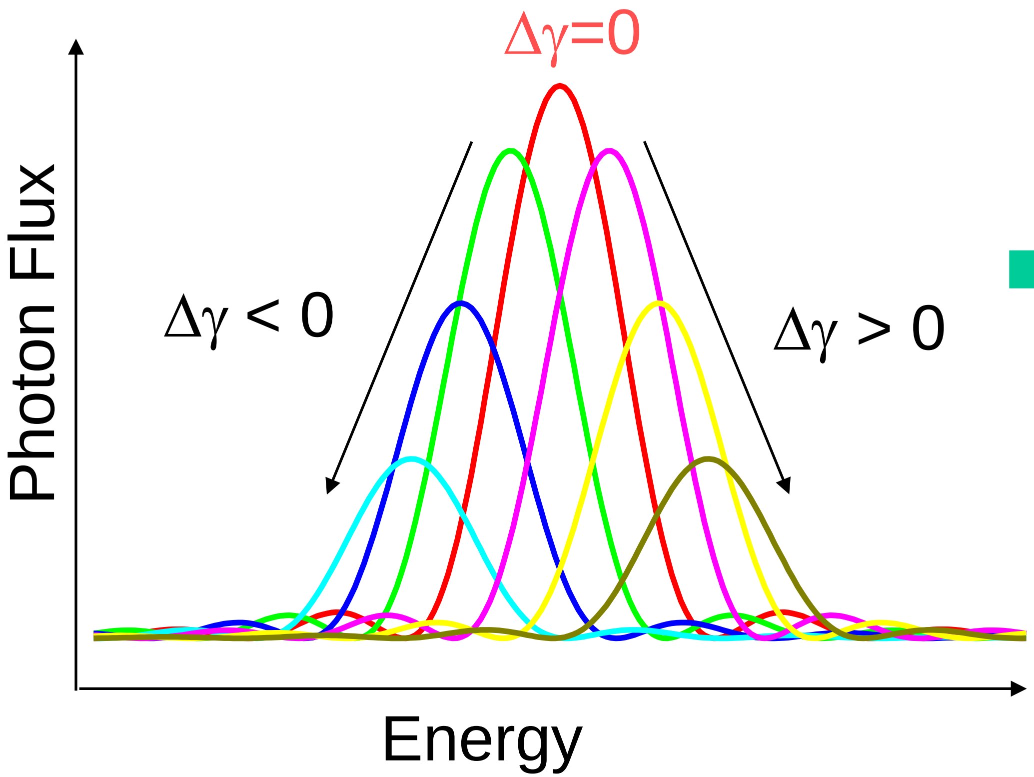


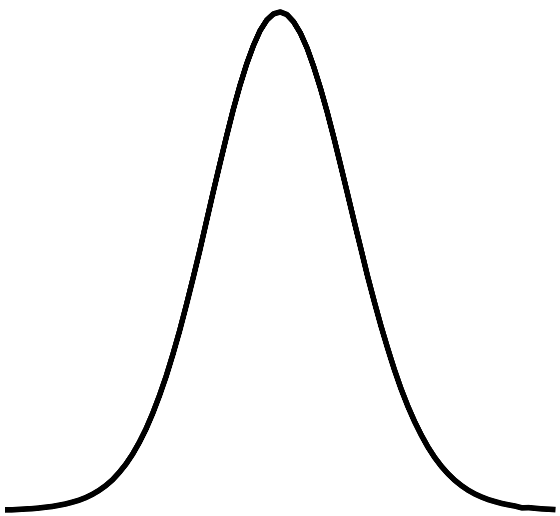
Effects due to the Energy Spread

Electron with an offset of $\Delta\gamma$

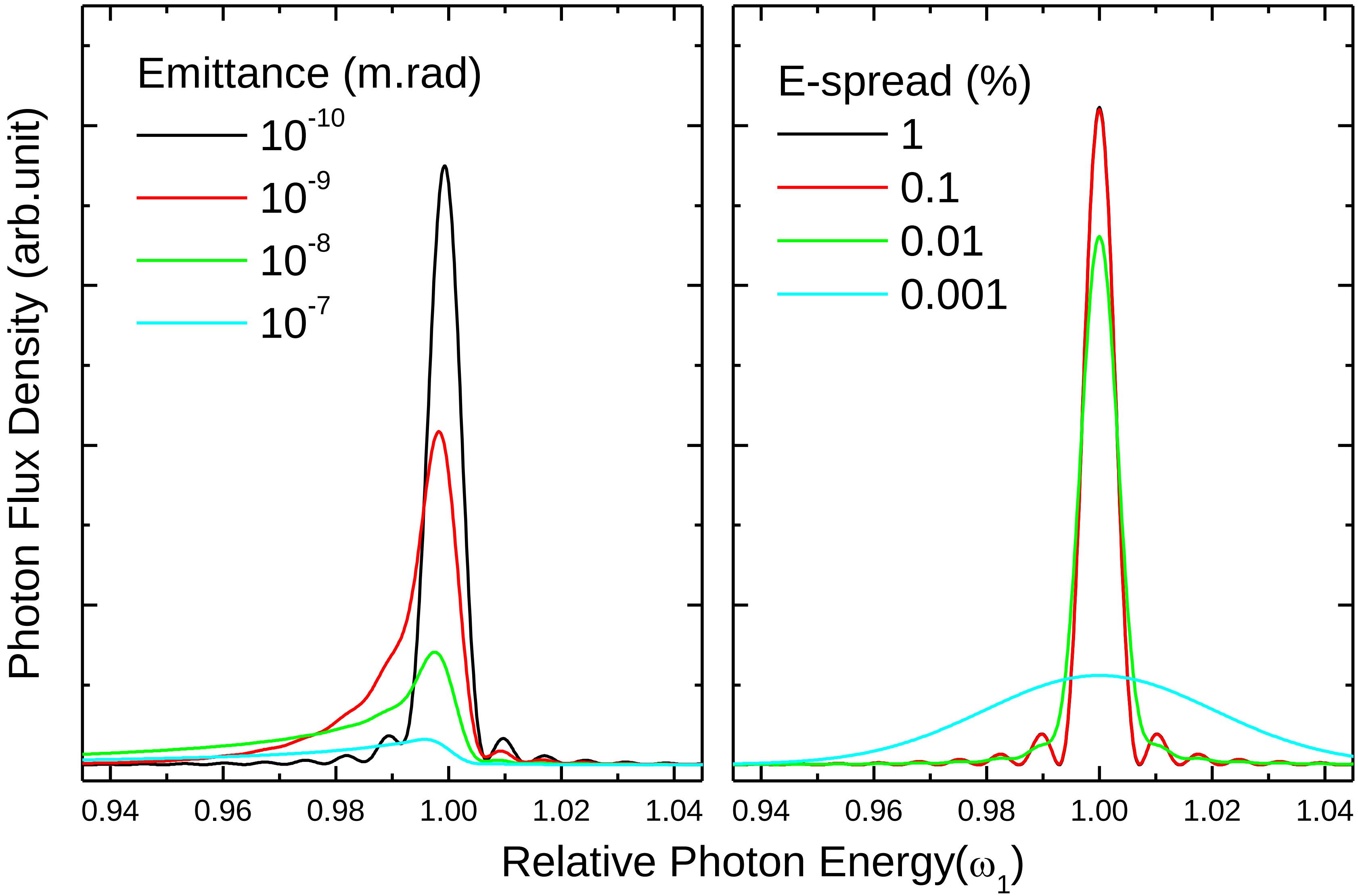
➡ Energy shift of ω_1

$$\omega_1(\gamma) = \frac{4\pi c \boxed{\gamma^2}/\lambda_u}{1 + K^2/2}$$





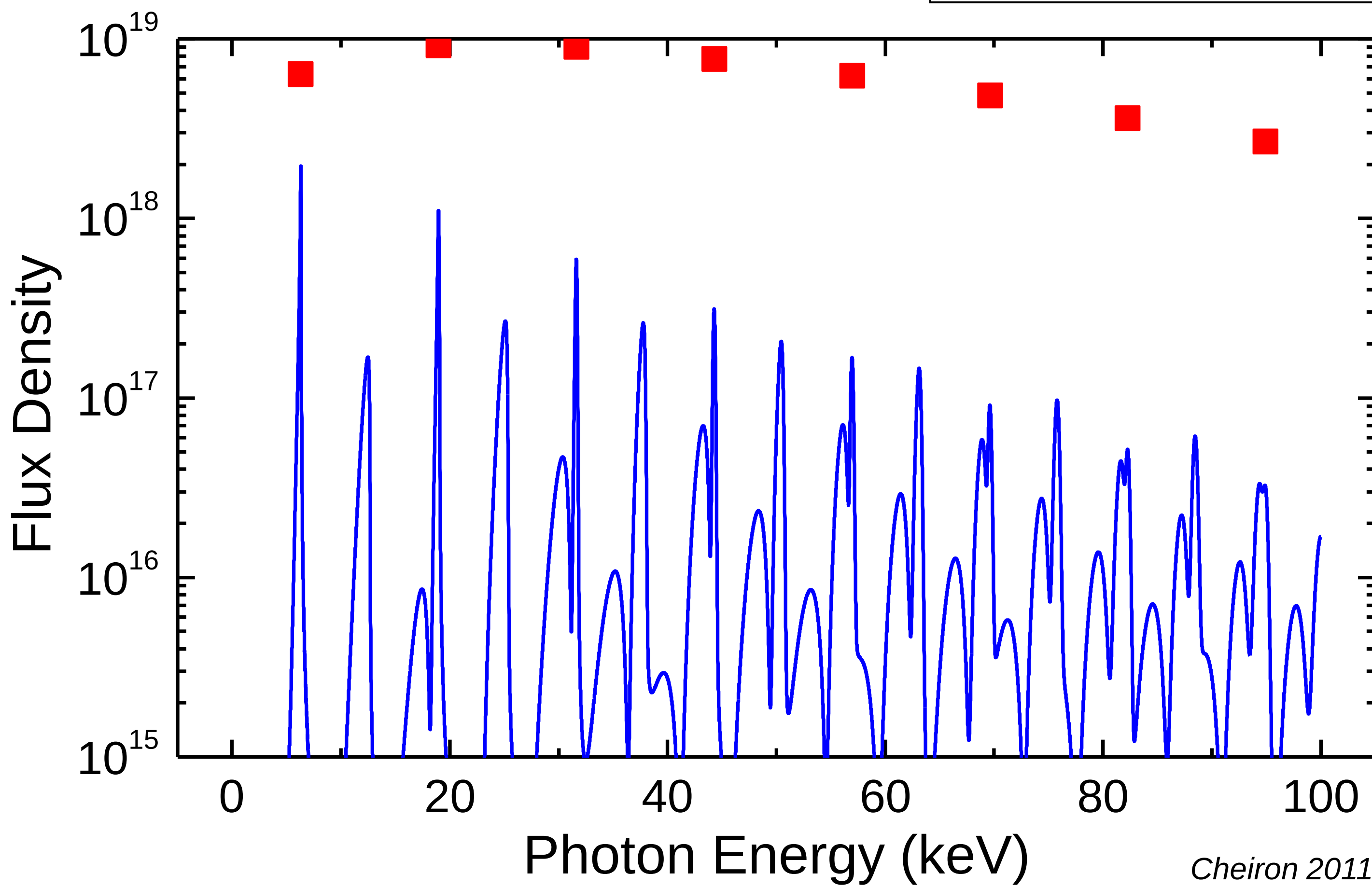
Examples


Photon Flux Density (arb.unit)
Emittance (m.rad)
10^{-10}
10^{-9}
10^{-8}
10^{-7}
E-spread (%)
1
0.1
0.01
0.001
0.94
0.96
0.98
1.00
1.02
1.04
Relative Photon Energy(ω_1)

Effects on the Higher Harmonics

Optical Emittance of UR: $\lambda/4\pi$
Bandwidth of UR: $\sim 1/nN$

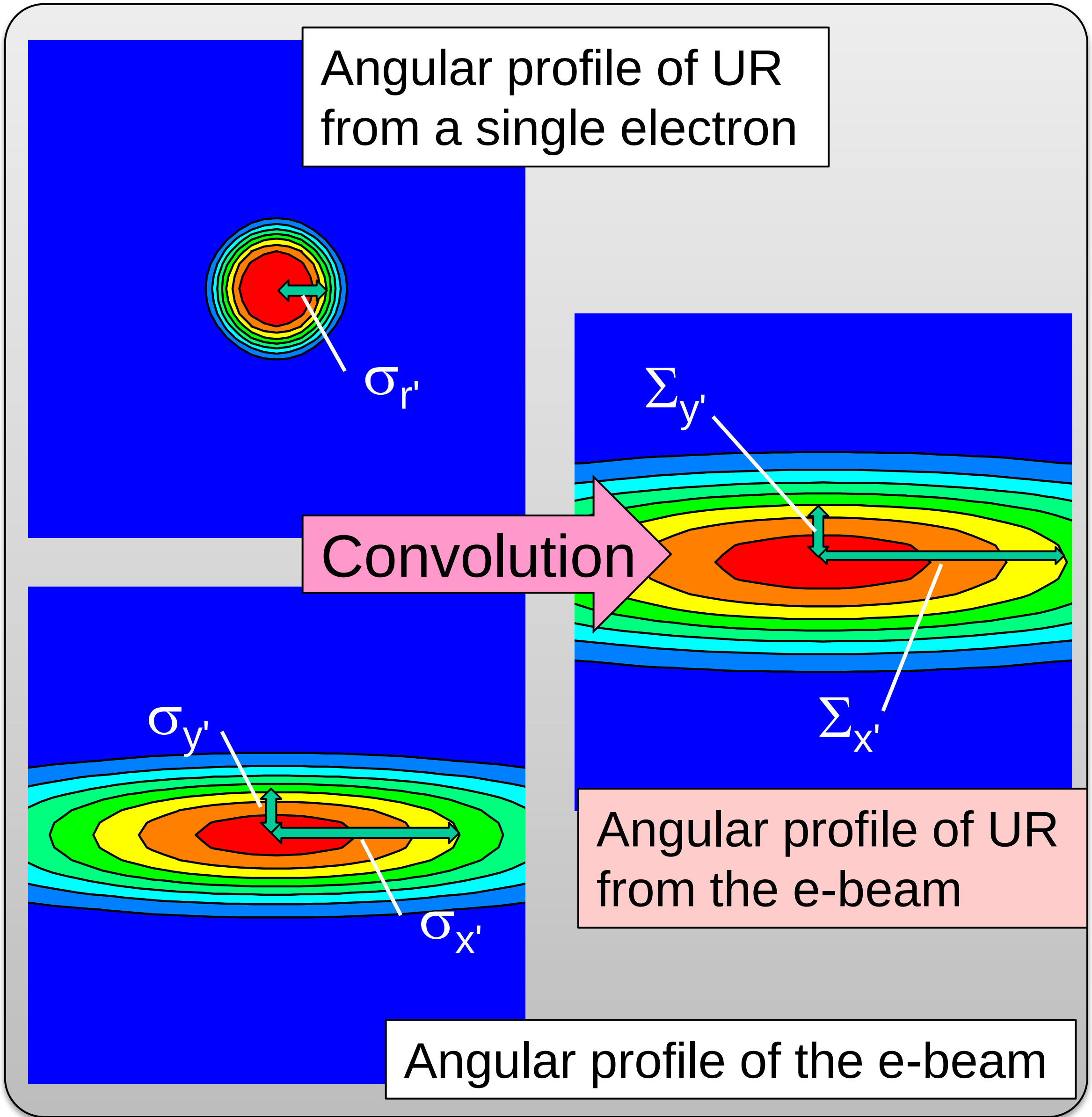
Effects due to the e^- beam are larger for higher harmonics



Evaluation of Optical Properties

- Optical properties of UR from the e-beam:
  - Calculate the UR from a single electron based on the theory of electrodynamics.
  - Integrate over the electrons in the beam (convolution)
    - Requires complicated numerical computation with a large number of parameters
    - Dedicated computer software (SPECTRA, SRW,…) is available
- Easy evaluation with Gauss approxmation
  - Source size, angular divergence
  - Flux density, brilliance

Effective Size & Divergence



By Gauss approx. and convolution theorem,

$$\Sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{x',y'}^2}$$

Effective Angular Div.

$$\Sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{x,y}^2}$$

Effective Source Size

Effective Flux Density and Brilliance

$$\int_{-\infty}^{\infty} G\exp(-x^2/2\sigma^2)dx = G \times \sqrt{2\pi}\sigma$$

Effective width of a Gauss function is $\sqrt{2\pi}\,\sigma$

Total Flux

$$F = \boxed{\left.\frac{d^2F}{dx'dy'}\right|_0} \times 2\pi\sigma_{r'}^2$$

on-axis flux density with zero-emittance beam

Effective Flux Density

$$\left.\frac{d^2F}{dx'dy'}\right|_e = \frac{F}{2\pi\Sigma_{x'}\Sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{\Sigma_{x'}\Sigma_{y'}}$$

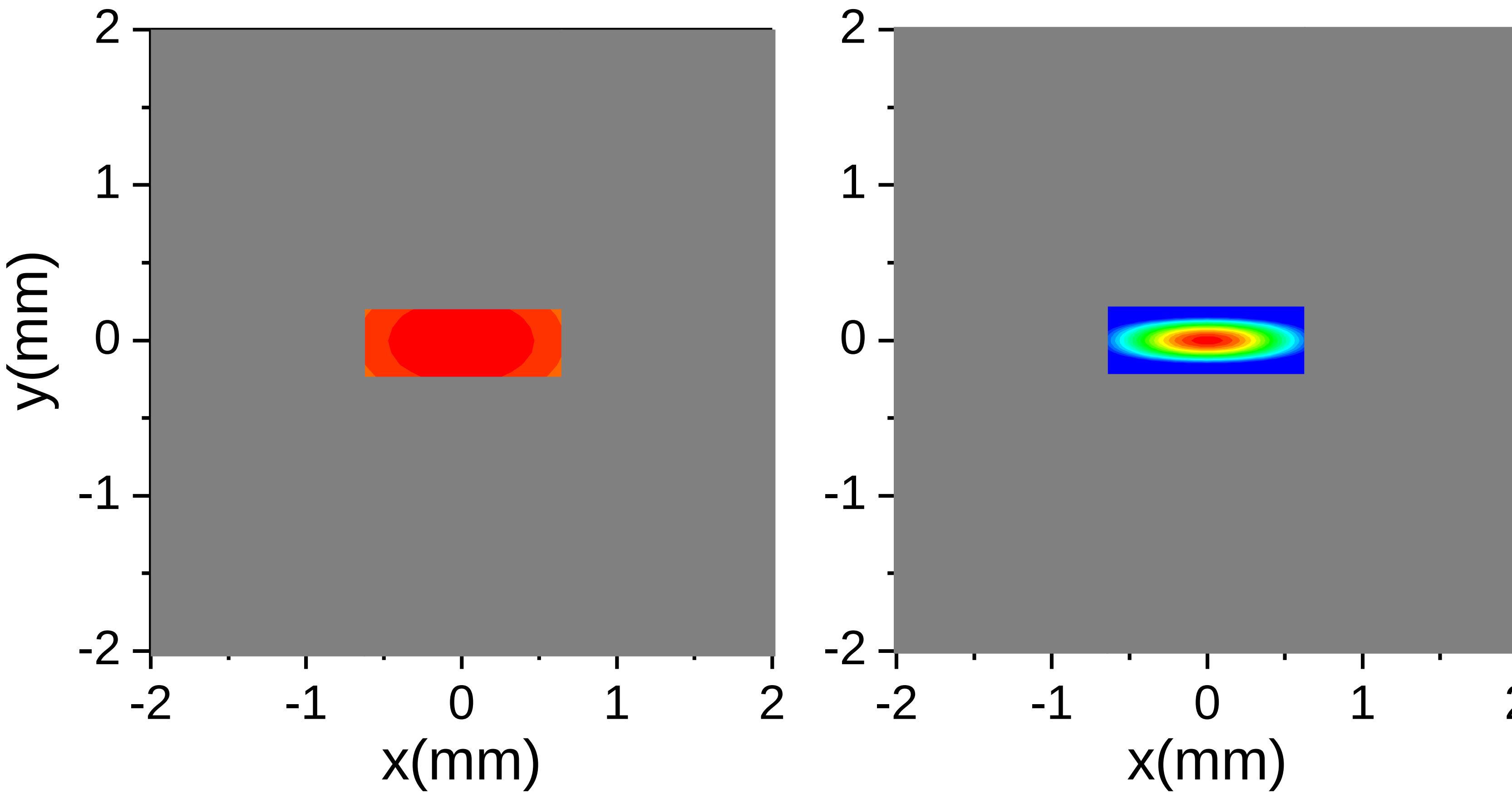
Effective Brilliance

$$B_e = \frac{F}{4\pi^2\Sigma_x\Sigma_y\Sigma_{x'}\Sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{2\pi\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}}$$

Heat Load on Optical Elements

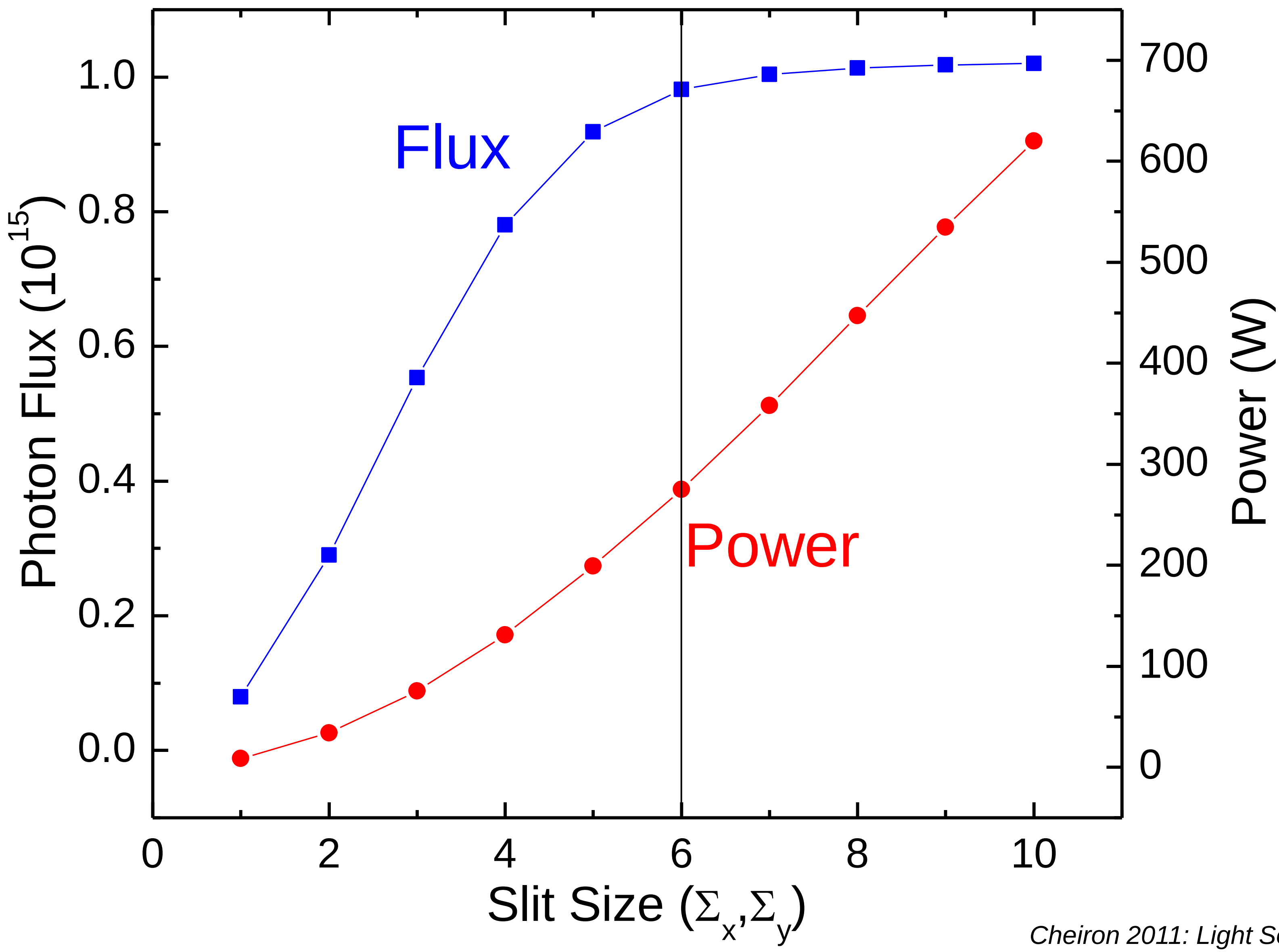
- SR emitted from the light source is processed by several optical elements before irradiation to the sample, such as the focusing mirror, monochromator.
- These elements can be easily damaged by the heat load brought by the SR.
- It is thus important to reduce the heat load as much as possible without sacrificing the flux by means of the XY slit at the front-end section.

Spatial Profile of Power and Flux



The power profile is much broader than the flux. Extraction of SR with an appropriate slit significantly reduces the heat load.

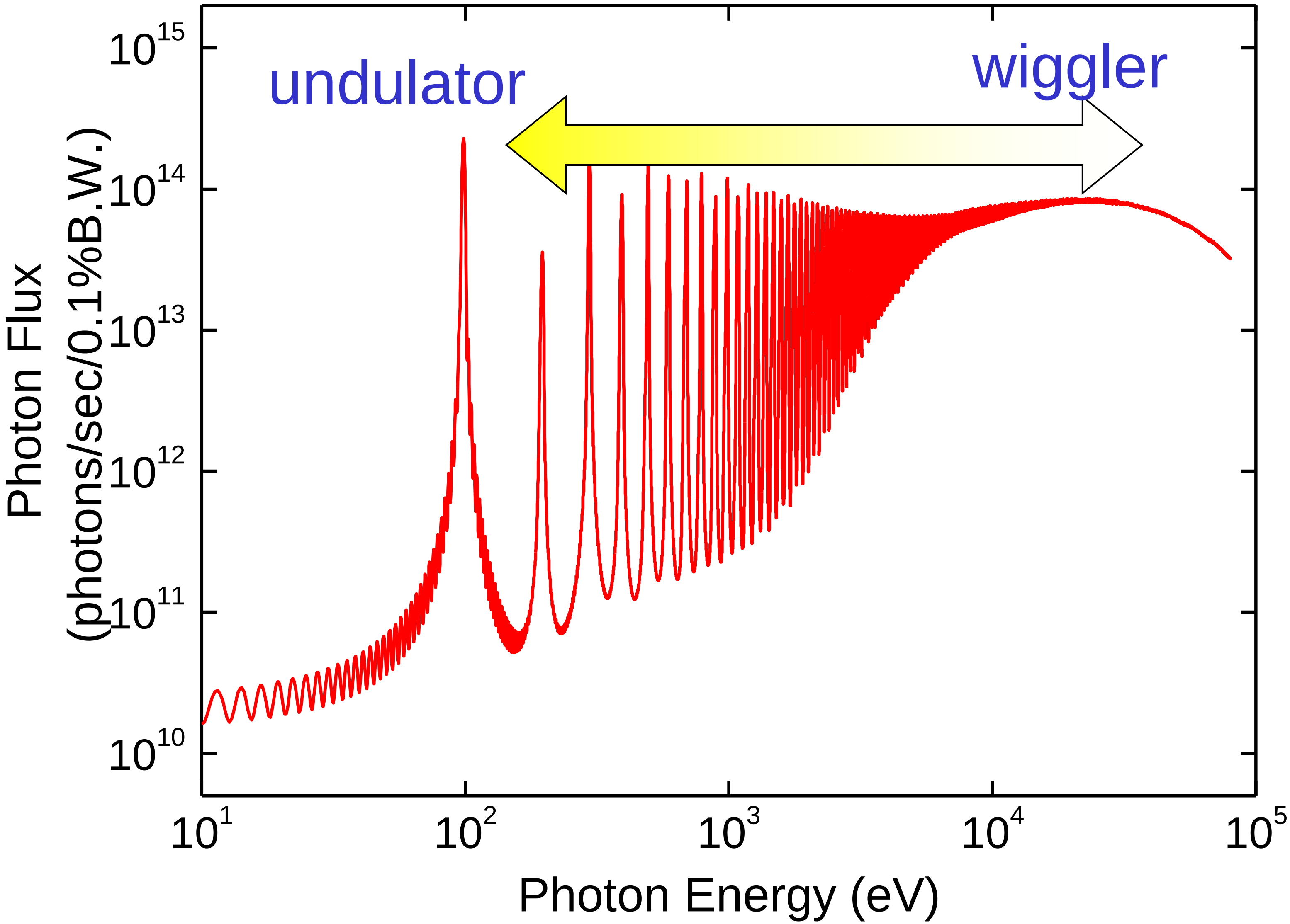
Optimum Slit Size?



Wiggler? Undulator? (1)

- Wigglers are identical to undulator from the point of view of magnetic circuit.
- It is generally said that the K value distinguishes between the two, however, this is not exactly correct.
- What we should take care is the region of photon energy to be utilized for application.

Wiggler? Undulator? (2)

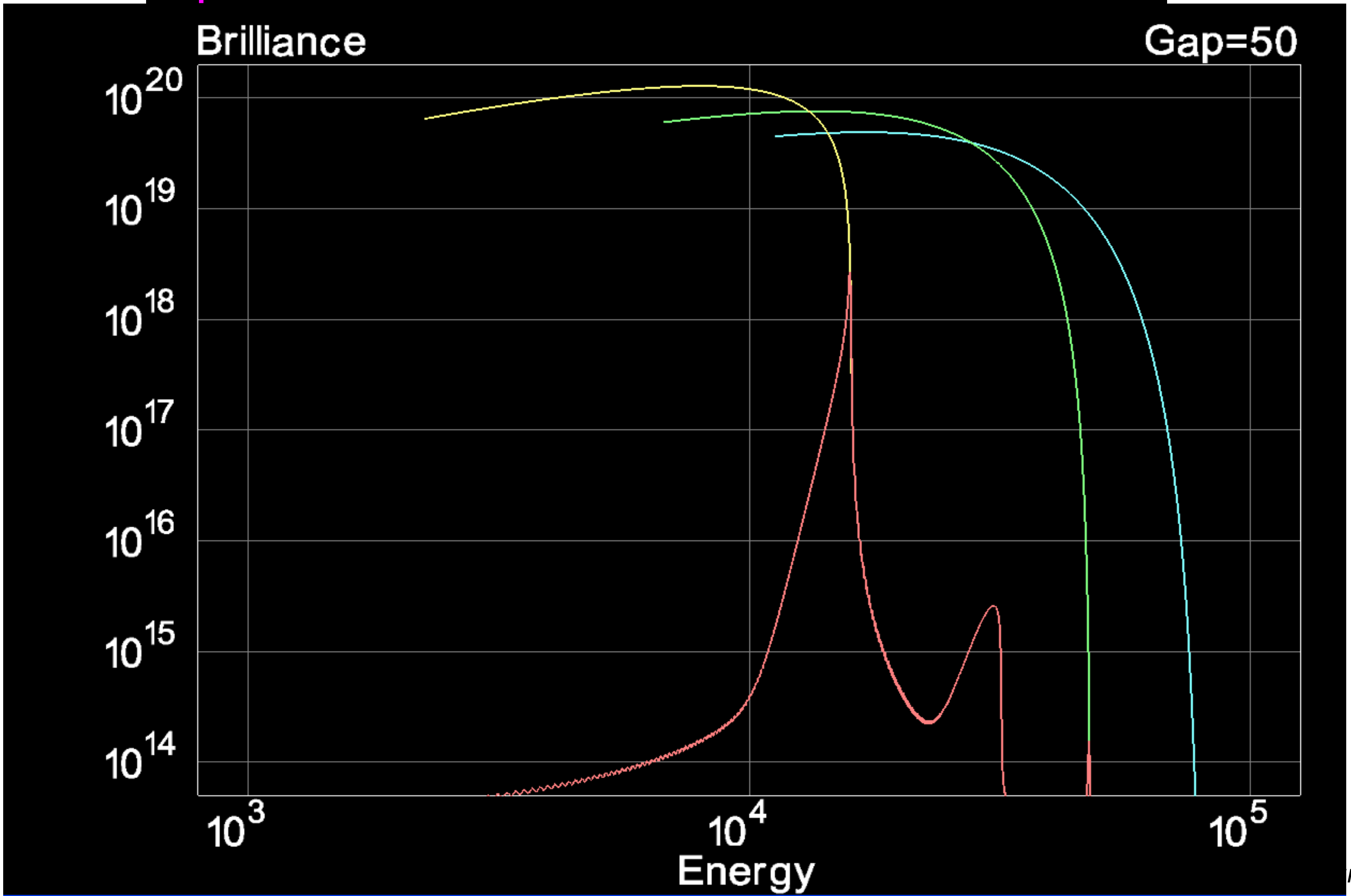


Undulator Radiation Gallery

- For quantitative evaluation of SR, a computer code “SPECTRA”, which has been developed and maintained in SPring-8 is available.
- SPECTRA also offers a function to “visualize” the computation results for further understanding of SR.
  - brilliance curve & spectrum
  - on- and off-peak angular profiles of flux
  - on- and off-axis spectra
  - effects of opening the slit aperture
  - undulator-to-wiggler transition

Brilliance Curve & Spectrum

Spectrum –, Peak Brilliance 1st – 3rd – 5th –



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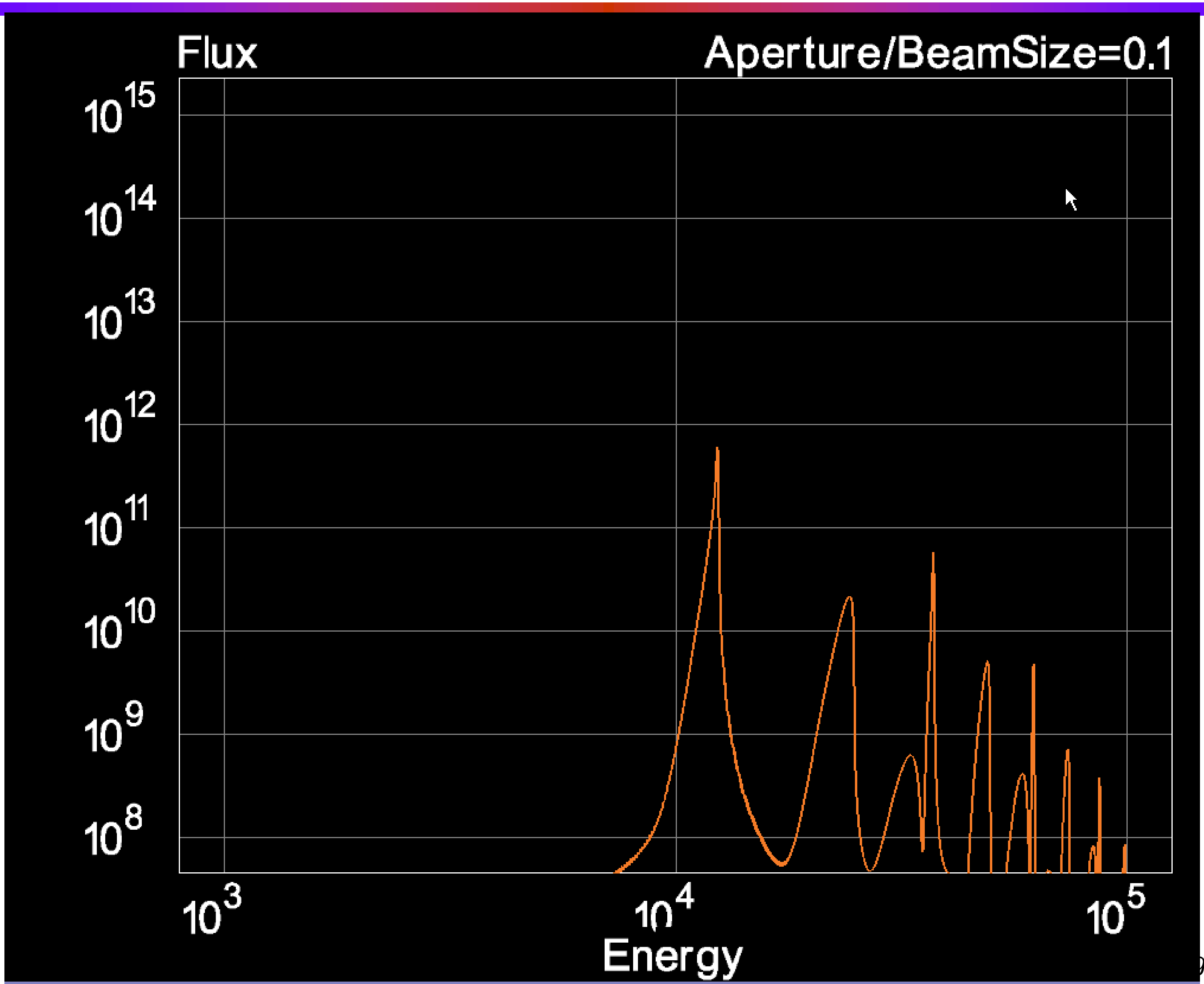
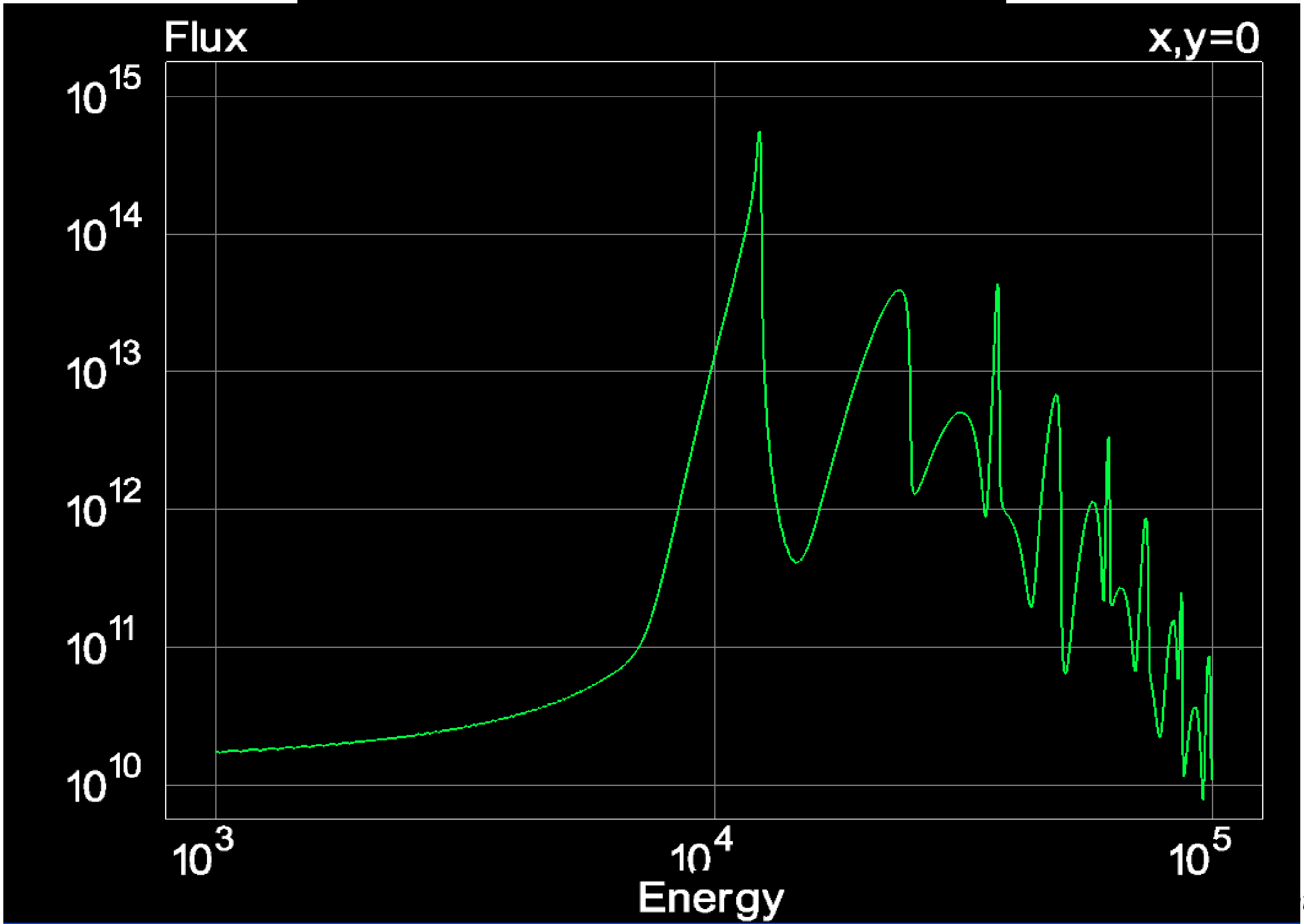
Opening the Slit Aperture



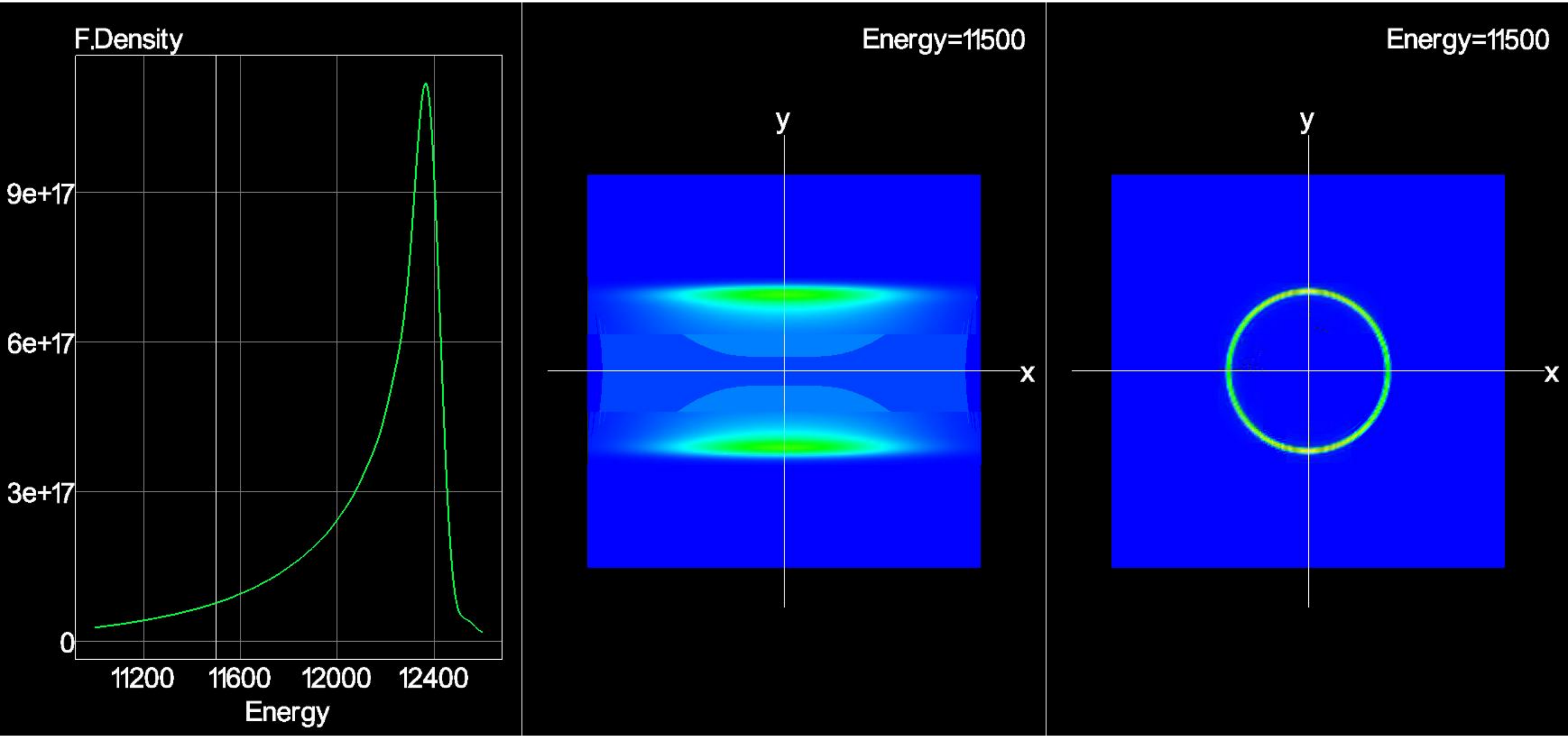
Off-Axis Spectrum

Moving the slit along – x, – y

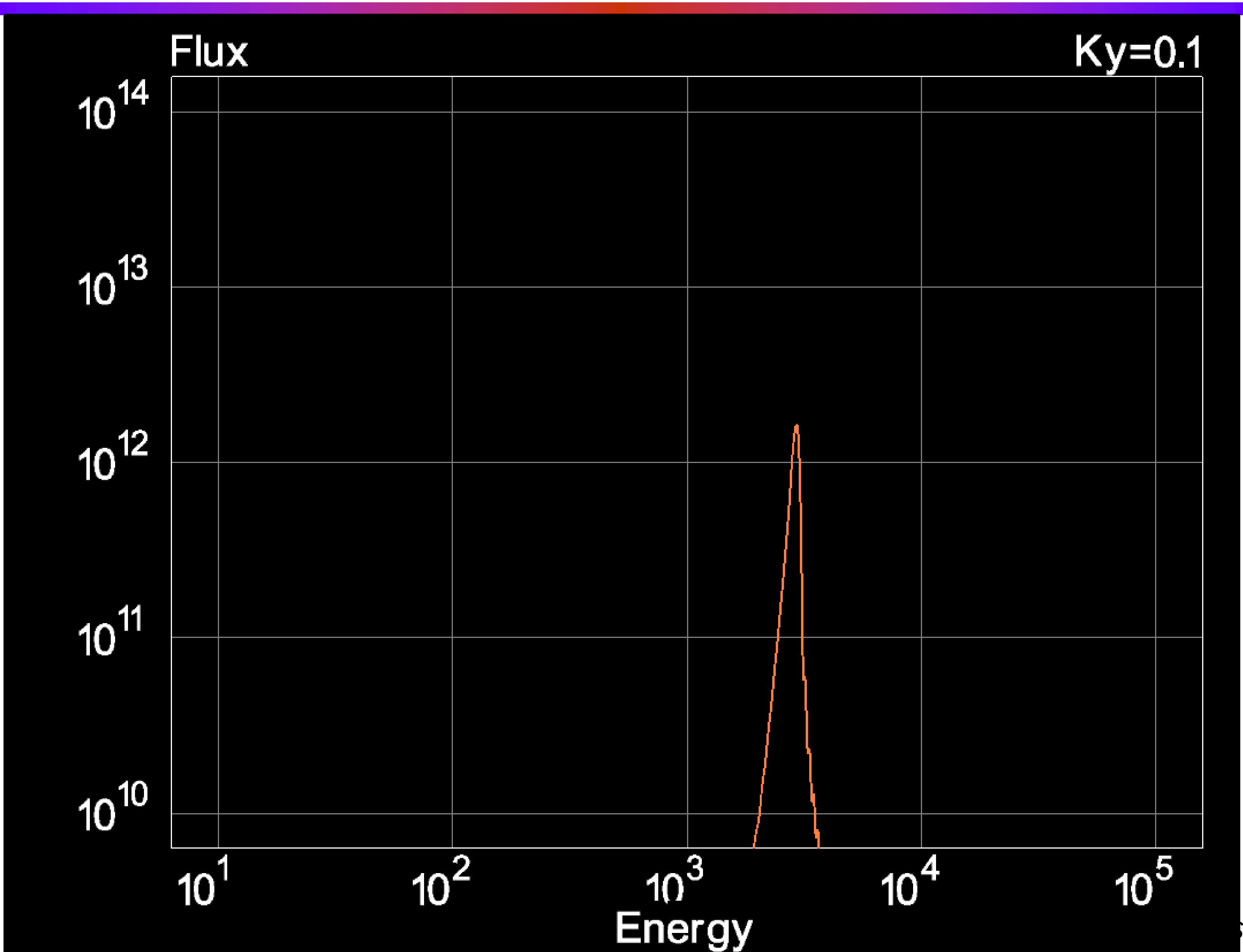


Flux Angular Profile

On-Axis Spectrum | Angular Profile (Finite Emittance) | Angular Profile (Zero Emittance)



Undulator-to-Wiggler Transition



Other Topics Not Addressed

- Quantitative descriptions of SR
- Light sources for circular polarization and schemes for fast helicity switching
  - helical undulator & elliptic wiggler
  - chicanes&choppers, kicker magnets
- Effects on the electron beam
  - natural focusing
  - beam-axis fluctuation due to COD variation
- R&Ds toward shorter magnetic period
  - superconducting undulators
  - cryogenic permanent magnet undulators
- Coherent SR for intense THz light
- Undulators for SASE-based X-ray FEL